Managerial Economics & Business Strategy

Chapter 10
Game Theory: Inside Oligopoly

McGraw-Hill/Irwin

Overview

I. Introduction to Game Theory
II. Simultaneous-Move, One-Shot Games
III. Infinitely Repeated Games
IV. Finitely Repeated Games
V. Multistage Games

Game Environments

- Players' planned decisions are called strategies.
- Payoffs to players are the profits or losses resulting from strategies.
- Order of play is important:
  - Simultaneous-move game: each player makes decisions with knowledge of other players' decisions.
  - Sequential-move game: one player observes its rival's move prior to selecting a strategy.
- Frequency of rival interaction
  - One-shot game: game is played once.
  - Repeated game: game is played more than once; either a finite or infinite number of interactions.

Simultaneous-Move, One-Shot Games: Normal Form Game

- A Normal Form Game consists of:
  – Set of players $i \in \{1, 2, \ldots n\}$ where n is a finite number.
  – Each players strategy set or feasible actions consist of a finite number of strategies.
    - Player 1's strategies are $S_1 = \{a, b, c, \ldots\}$.
    - Player 2's strategies are $S_2 = \{A, B, C, \ldots\}$.
  – Payoffs.
    - Player 1's payoff: $\pi_1(a,B) = 11$.
    - Player 2's payoff: $\pi_2(b,C) = 12$.

A Normal Form Game

Player 2

Player 1

Strategy	A	B	C
a	12,11	11,12	14,13
b	11,10	10,11	12,12
c	10,15	10,13	13,14

Normal Form Game: Scenario Analysis

- Suppose 1 thinks 2 will choose “A”.

Player 2

Player 1

Strategy	A	B	C
a	12,11	11,12	14,13
b	11,10	10,11	12,12
c	10,15	10,13	13,14

Normal Form Game: Scenario Analysis

- Then 1 should choose "a".
  - Player 1's best response to "A" is "a".

Player 2

Player 1

Strategy	A	B	C
a	12,11	11,12	14,13
b	11,10	10,11	12,12
c	10,15	10,13	13,14

Normal Form Game: Scenario Analysis

- Suppose 1 thinks 2 will choose “B”.

Player 2

Player 1

Strategy	A	B	C
a	12,11	11,12	14,13
b	11,10	10,11	12,12
c	10,15	10,13	13,14

Normal Form Game: Scenario Analysis

- Then 1 should choose "a".
  - Player 1's best response to "B" is "a".

Player 2

Player 1

Strategy	A	B	C
a	12,11	11,12	14,13
b	11,10	10,11	12,12
c	10,15	10,13	13,14

Normal Form Game
Scenario Analysis

- Similarly, if 1 thinks 2 will choose C….
  - Player 1's best response to "C" is "a".

Player 2

Player 1

Strategy	A	B	C
a	12,11	11,12	14,13
b	11,10	10,11	12,12
c	10,15	10,13	13,14

Dominant Strategy

- Regardless of whether Player 2 chooses A, B, or C, Player 1 is better off choosing “a”!
- “a” is Player 1’s Dominant Strategy!

Player 2

Player 1

Strategy	A	B	C
a	12,11	11,12	14,13
b	11,10	10,11	12,12
c	10,15	10,13	13,14

Dominant Strategy in a Simultaneous-Move, One-Shot Game

- A dominant strategy is a strategy resulting in the highest payoff regardless of the opponent's action.
- If "a" is a dominant strategy for Player 1 in the previous game, then:
  – $\pi_1(a,A) > \pi_1(b,A) \geq \pi_1(c,A)$;
  – $\pi_1(a,B) > \pi_1(b,B) \geq \pi_1(c,B)$;
  – and $\pi_1(a,C) > \pi_1(b,C) \geq \pi_1(c,C)$.

Putting Yourself in your Rival's Shoes

- What should player 2 do?
  - 2 has no dominant strategy!
  - But 2 should reason that 1 will play "a".
  - Therefore 2 should choose "C".

Player 2

Player 1

Strategy	A	B	C
a	12,11	11,12	14,13
b	11,10	10,11	12,12
c	10,15	10,13	13,14

The Outcome

Player 2

Player 1

Strategy	A	B	C
a	12,11	11,12	14,13
b	11,10	10,11	12,12
c	10,15	10,13	13,14

- This outcome is called a Nash equilibrium:
  - “a” is player 1’s best response to “C”.
  - “C” is player 2’s best response to “a”.

Two-Player Nash Equilibrium

- The Nash equilibrium is a condition describing the set of strategies in which no player can improve her payoff by unilaterally changing her own strategy, given the other player's strategy.
- Formally,
  - $\pi_1(s_1^*, s_2^*) \geq \pi_1(s_1, s_2^*)$ for all s_1.
  - $\pi_1(s_1^*, s_2^*) \geq \pi_1(s_1^*, s_2)$ for all s_2.

Key Insights

- Look for dominant strategies.
- Put yourself in your rival's shoes.

A Market-Share Game

- Two managers want to maximize market share: $i \in \{1,2\}$.
- Strategies are pricing decisions
  - $S_1 = \{1, 5, 10\}$.
  - $S_2 = \{1, 5, 10\}$.
- Simultaneous moves.
- One-shot game.

The Market-Share Game in Normal Form

Manager 2

Manager 1

Strategy	P=$10	P=$5	P = $1
P=$10	.5, .5	.2, .8	.1, .9
P=$5	.8, .2	.5, .5	.2, .8
P=$1	.9, .1	.8, .2	.5, .5

Market-Share Game Equilibrium

Manager 2

Manager 1

Strategy	P=$10	P=$5	P = $1
P=$10	.5, .5	.2, .8	.1, .9
P=$5	.8, .2	.5, .5	.2, .8
P=$1	.9, .1	.8, .2	.5, .5

Nash Equilibrium

Key Insight:

- Game theory can be used to analyze situations where “payoffs” are non monetary!
- We will, without loss of generality, focus on environments where businesses want to maximize profits.
  - Hence, payoffs are measured in monetary units.

Coordination Games

- In many games, players have competing objectives: One firm gains at the expense of its rivals.
- However, some games result in higher profits by each firm when they "coordinate" decisions.

Examples of Coordination Games

- Industry standards
  - size of floppy disks.
  - size of CDs.
- National standards
  - electric current.
  - traffic laws.

A Coordination Game in Normal Form

Player 2

Player 1

Strategy	A	B	C
1	0,0	0,0	$10,$10
2	$10,$10	0,0	0,0
3	0,0	$10,$10	0,0

A Coordination Problem: Three Nash Equilibria!

Player 2

Player 1

Strategy	A	B	C
1	0,0	0,0	$10,$10
2	$10,$10	0,0	0,0
3	0,0	$10, $10	0,0

Key Insights:

- Not all games are games of conflict.
- Communication can help solve coordination problems.
- Sequential moves can help solve coordination problems.

Games With No Pure Strategy Nash Equilibrium

Player 2

Player 1

Strategy	A	B
1	-100,100	100,-100
2	100,-100	-100,100

Strategies for Games With No Pure Strategy Nash Equilibrium

- In games where no pure strategy Nash equilibrium exists, players find it in there interest to engage in mixed (randomized) strategies.
  - This means players will "randomly" select strategies from all available strategies.

An Advertising Game

- Two firms (Kellogg's & General Mills) managers want to maximize profits.
- Strategies consist of advertising campaigns.
- Simultaneous moves.
  – One-shot interaction.
  – Repeated interaction.

A One-Shot Advertising Game

General Mills

Kellogg's

Strategy	None	Moderate	High
None	12,12	1, 20	-1, 15
Moderate	20, 1	6, 6	0, 9
High	15, -1	9, 0	2, 2

Equilibrium to the One-Shot Advertising Game

General Mills

Kellogg's

Strategy	None	Moderate	High
None	12,12	1, 20	-1, 15
Moderate	20, 1	6, 6	0, 9
High	15, -1	9, 0	2, 2

Nash Equilibrium

Can collusion work if the game is repeated 2 times?

General Mills

Kellogg's

Strategy	None	Moderate	High
None	12,12	1, 20	-1, 15
Moderate	20, 1	6, 6	0, 9
High	15, -1	9, 0	2, 2

No (by backwards induction).

- In period 2, the game is a one-shot game, so equilibrium entails High Advertising in the last period.
- This means period 1 is "really" the last period, since everyone knows what will happen in period 2.
- Equilibrium entails High Advertising by each firm in both periods.
- The same holds true if we repeat the game any known, finite number of times.

Can collusion work if firms play the game each year, forever?

- Consider the following “trigger strategy” by each firm:
  - “Don’t advertise, provided the rival has not advertised in the past. If the rival ever advertises, “punish” it by engaging in a high level of advertising forever after.”
- In effect, each firm agrees to “cooperate” so long as the rival hasn’t “cheated” in the past. “Cheating” triggers punishment in all future periods.

Suppose General Mills adopts this trigger strategy. Kellogg's profits?

$\Pi_{Cooperate} = 12 + 12/(1+i) + 12/(1+i)^2 + 12/(1+i)^3 + \ldots$

$= 12 + \boxed{12/i}$ ← Value of a perpetuity of $12 paid at the end of every year

$\Pi_{Cheat} = 20 + 2/(1+i) + 2/(1+i)^2 + 2/(1+i)^3 + \ldots$

$= 20 + 2/i$

General Mills

Kellogg's

Strategy	None	Moderate	High
None	12,12	1, 20	-1, 15
Moderate	20, 1	6, 6	0, 9
High	15, -1	9, 0	2, 2

Kellogg's Gain to Cheating:

- $\Pi_{Cheat} - \Pi_{Cooperate} = 20 + 2/i - (12 + 12/i) = 8 - 10/i$
  – Suppose i = .05
- $\Pi_{Cheat} - \Pi_{Cooperate} = 8 - 10/.05 = 8 - 200 = -192$
- It doesn't pay to deviate.
  – Collusion is a Nash equilibrium in the infinitely repeated game!

General Mills

Kellogg's

Strategy	None	Moderate	High
None	12,12	1, 20	-1, 15
Moderate	20, 1	6, 6	0, 9
High	15, -1	9, 0	2, 2

Benefits & Costs of Cheating

- $\Pi_{Cheat} - \Pi_{Cooperate} = 8 - 10/i$
  - 8 = Immediate Benefit (20 - 12 today)
  - 10/i = PV of Future Cost (12 - 2 forever after)
- If Immediate Benefit - PV of Future Cost > 0
  - Pays to "cheat".
- If Immediate Benefit - PV of Future Cost ≤ 0
  - Doesn't pay to "cheat".

General Mills (columns); **Kellogg's** (rows)

Strategy	None	Moderate	High
None	12,12	1, 20	-1, 15
Moderate	20, 1	6, 6	0, 9
High	15, -1	9, 0	2, 2

Key Insight

- Collusion can be sustained as a Nash equilibrium when there is no certain “end” to a game.
- Doing so requires:
  - Ability to monitor actions of rivals.
  - Ability (and reputation for) punishing defectors.
  - Low interest rate.
  - High probability of future interaction.

Real World Examples of Collusion

- Garbage Collection Industry
- OPEC
- NASDAQ
- Airlines
- Lysine Market

Normal-Form Bertrand Game

Firm 2

Firm 1

Strategy	Low Price	High Price
Low Price	0,0	20,-1
High Price	-1, 20	15, 15

One-Shot Bertrand (Nash) Equilibrium

Firm 2

Firm 1

Strategy	Low Price	High Price
Low Price	0,0	20,-1
High Price	-1, 20	15, 15

Potential Repeated Game Equilibrium Outcome

Firm 2

Firm 1

Strategy	Low Price	High Price
Low Price	0,0	20,-1
High Price	-1, 20	15, 15

Simultaneous-Move Bargaining

- Management and a union are negotiating a wage increase.
- Strategies are wage offers & wage demands.
- Successful negotiations lead to $600 million in surplus, which must be split among the parties.
- Failure to reach an agreement results in a loss to the firm of $100 million and a union loss of $3 million.
- Simultaneous moves, and time permits only one-shot at making a deal.

The Bargaining Game in Normal Form

Union

Management

Strategy	W = $10	W = $5	W = $1
W = $10	100, 500	-100, -3	-100, -3
W=$5	-100, -3	300, 300	-100, -3
W=$1	-100, -3	-100, -3	500, 100

Three Nash Equilibria!

Union

Management

Strategy	W = $10	W = $5	W = $1
W = $10	100, 500	-100, -3	-100, -3
W=$5	-100, -3	300, 300	-100, -3
W=$1	-100, -3	-100, -3	500, 100

Fairness: The “Natural” Focal Point

Union

Management

Strategy	W = $10	W = $5	W = $1
W = $10	100, 500	-100, -3	-100, -3
W=$5	-100, -3	300, 300	-100, -3
W=$1	-100, -3	-100, -3	500, 100

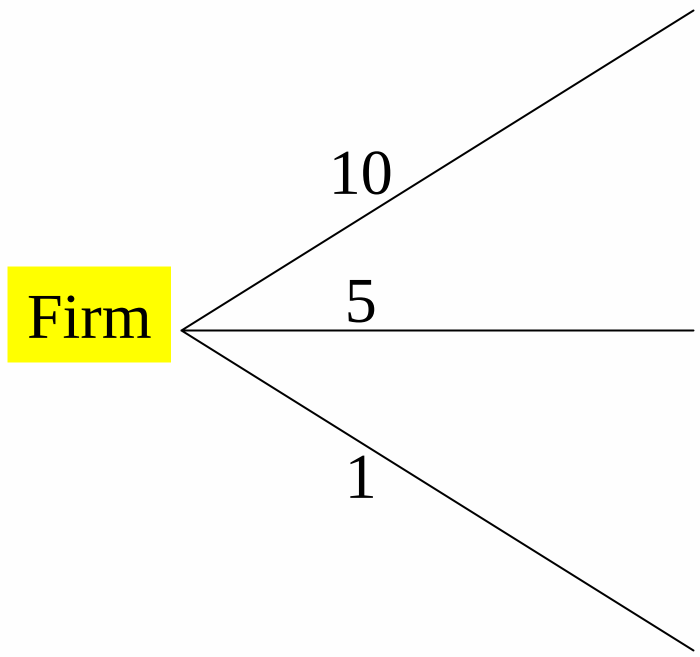
Lessons in Simultaneous Bargaining

- Simultaneous-move bargaining results in a coordination problem.
- Experiments suggests that, in the absence of any “history,” real players typically coordinate on the “fair outcome.”
- When there is a “bargaining history,” other outcomes may prevail.

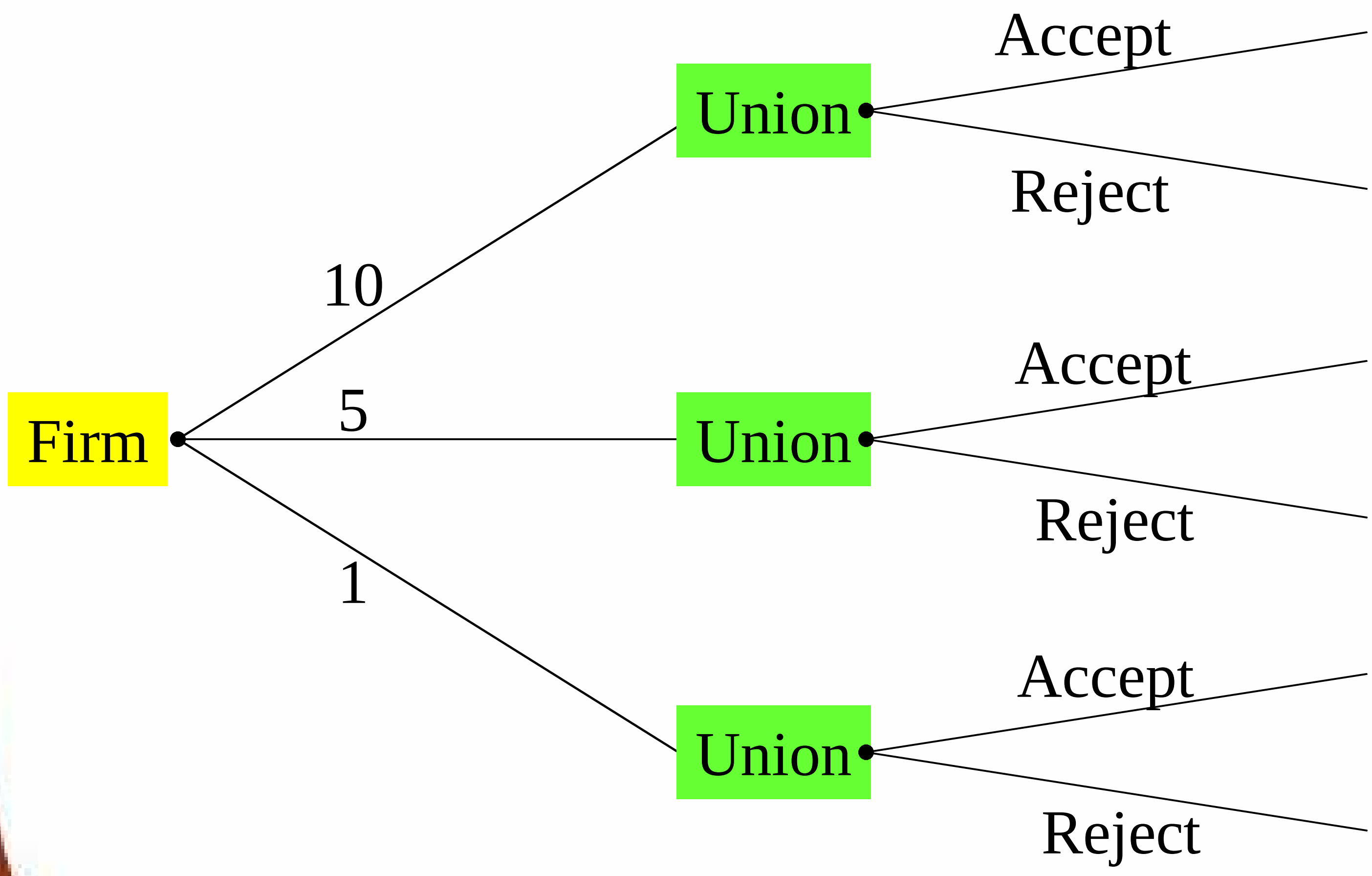
Single-Offer Bargaining

- Now suppose the game is sequential in nature, and management gets to make the union a "take-it-or-leave-it" offer.
- Analysis Tool: Write the game in extensive form
  - Summarize the players.
  - Their potential actions.
  - Their information at each decision point.
  - Sequence of moves.
  - Each player's payoff.

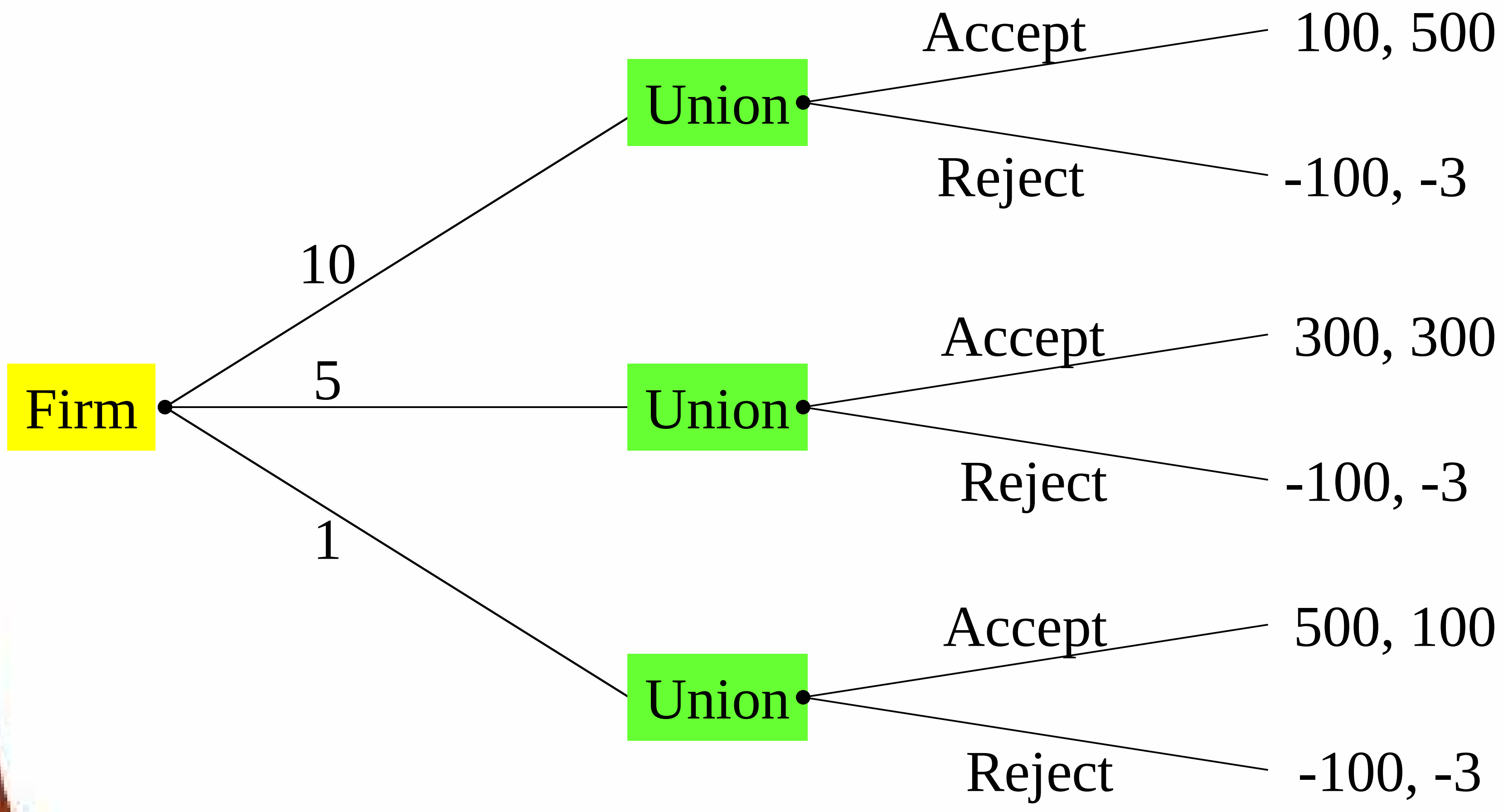
Step 1: Management's Move



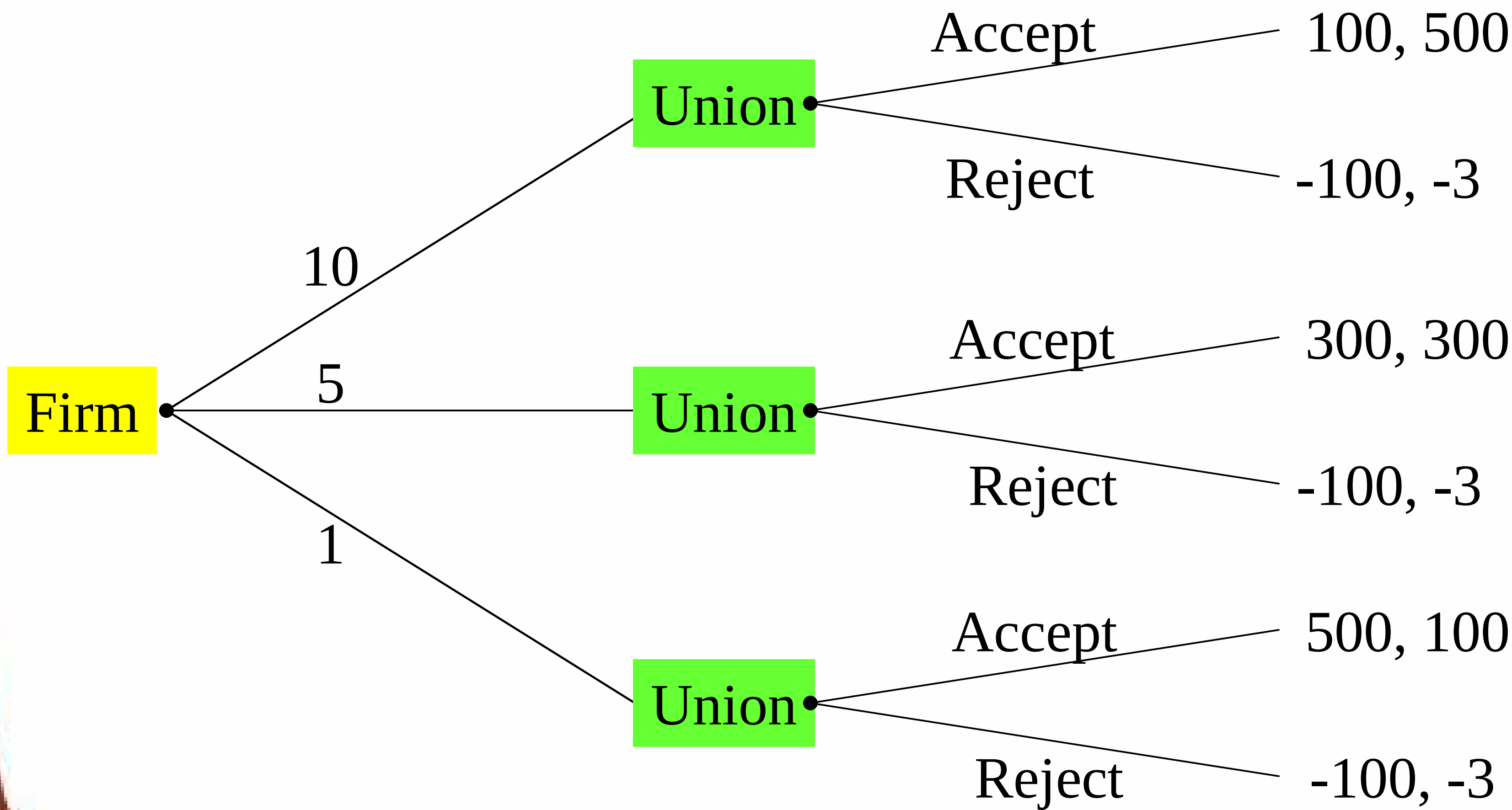
Step 2: Add the Union's Move



Step 3: Add the Payoffs



The Game in Extensive Form



Step 4: Identify the Firm's Feasible Strategies

- Management has one information set and thus three feasible strategies:
  - – Offer $10.
  - – Offer $5.
  - – Offer $1.

Step 5: Identify the Union's Feasible Strategies

- The Union has three information set and thus eight feasible strategies ($2^3=8$):
  - Accept $10, Accept $5, Accept $1
  - Accept $10, Accept $5, Reject $1
  - Accept $10, Reject $5, Accept $1
  - Accept $10, Reject $5, Reject $1
  - Reject $10, Accept $5, Accept $1
  - Reject $10, Accept $5, Reject $1
  - Reject $10, Reject $5, Accept $1
  - Reject $10, Reject $5, Reject $1

Step 6: Identify Nash Equilibrium Outcomes

- Outcomes such that neither the firm nor the union has an incentive to change its strategy, given the strategy of the other.

Finding Nash Equilibrium Outcomes

Union's Strategy	Firm's Best Response	Mutual Best Response?
Accept $10, Accept $5, Accept $1	$1	Yes
Accept $10, Accept $5, Reject $1	$5	Yes
Accept $10, Reject $5, Accept $1	$1	Yes
Reject $10, Accept $5, Accept $1	$1	Yes
Accept $10, Reject $5, Reject $1	$10	Yes
Reject $10, Accept $5, Reject $1	$5	Yes
Reject $10, Reject $5, Accept $1	$1	Yes
Reject $10, Reject $5, Reject $1	$10, $5, $1	No

Step 7: Find the Subgame Perfect Nash Equilibrium Outcomes

- Outcomes where no player has an incentive to change its strategy, given the strategy of the rival, and
- The outcomes are based on “credible actions;” that is, they are not the result of “empty threats” by the rival.

Checking for Credible Actions

Union's Strategy	Are all Actions Credible?
Accept $10, Accept $5, Accept $1	Yes
Accept $10, Accept $5, Reject $1	No
Accept $10, Reject $5, Accept $1	No
Reject $10, Accept $5, Accept $1	No
Accept $10, Reject $5, Reject $1	No
Reject $10, Accept $5, Reject $1	No
Reject $10, Reject $5, Accept $1	No
Reject $10, Reject $5, Reject $1	No

The “Credible” Union Strategy

Union's Strategy	Are all Actions Credible?
Accept $10, Accept $5, Accept $1	Yes
Accept $10, Accept $5, Reject $1	No
Accept $10, Reject $5, Accept $1	No
Reject $10, Accept $5, Accept $1	No
Accept $10, Reject $5, Reject $1	No
Reject $10, Accept $5, Reject $1	No
Reject $10, Reject $5, Accept $1	No
Reject $10, Reject $5, Reject $1	No

Finding Subgame Perfect Nash Equilibrium Strategies

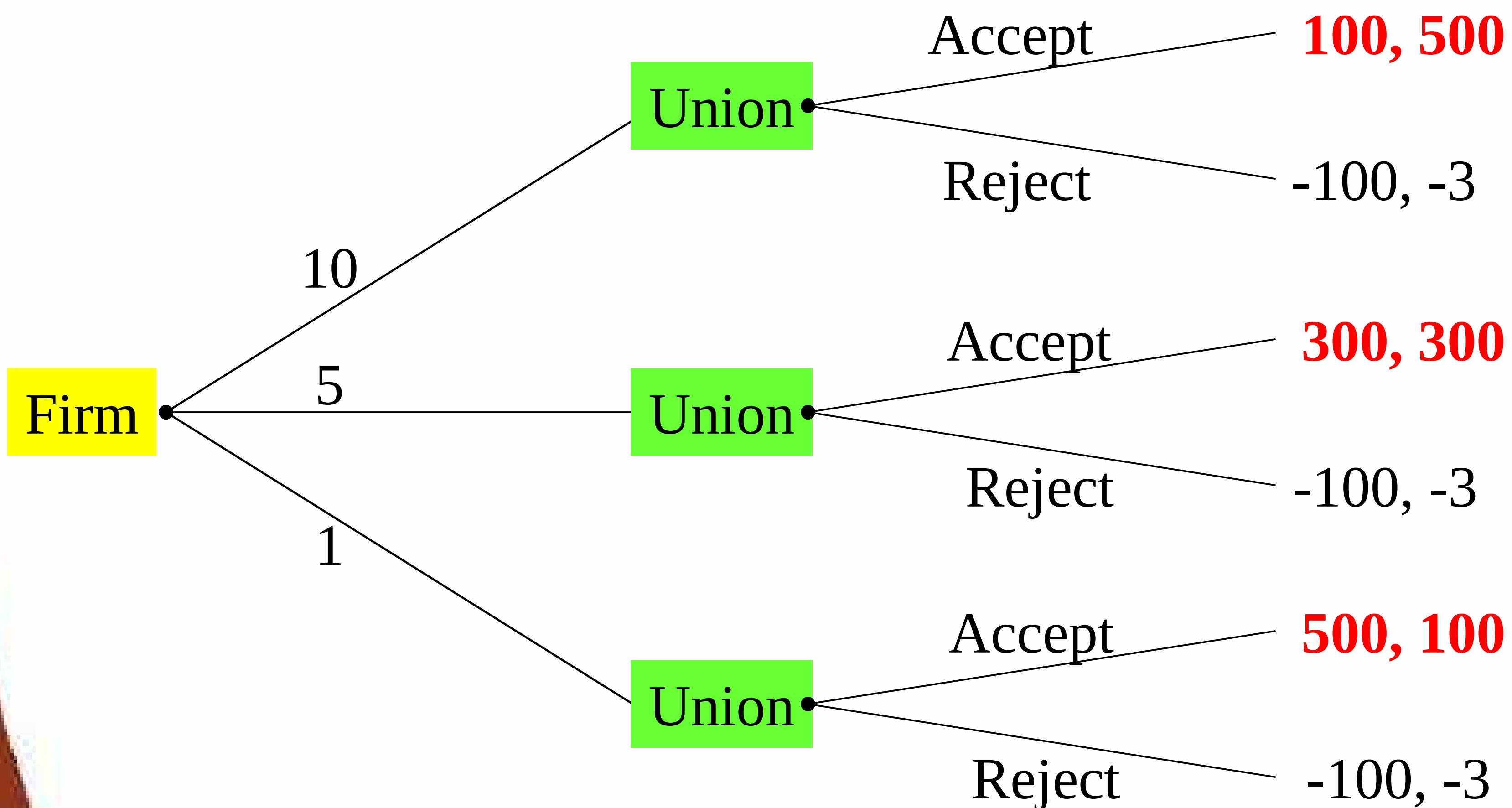
Union's Strategy	Firm's Best Response	Mutual Best Response?
Accept $10, Accept $5, Accept $1	$1	Yes
Accept $10, Accept $5, Reject $1	$5	Yes
Accept $10, Reject $5, Accept $1	$1	Yes
Reject $10, Accept $5, Accept $1	$1	Yes
Accept $10, Reject $5, Reject $1	$10	Yes
Reject $10, Accept $5, Reject $1	$5	Yes
Reject $10, Reject $5, Accept $1	$1	Yes
Reject $10, Reject $5, Reject $1	$10, $5, $1	No

Nash and Credible | Nash Only | Neither Nash Nor Credible

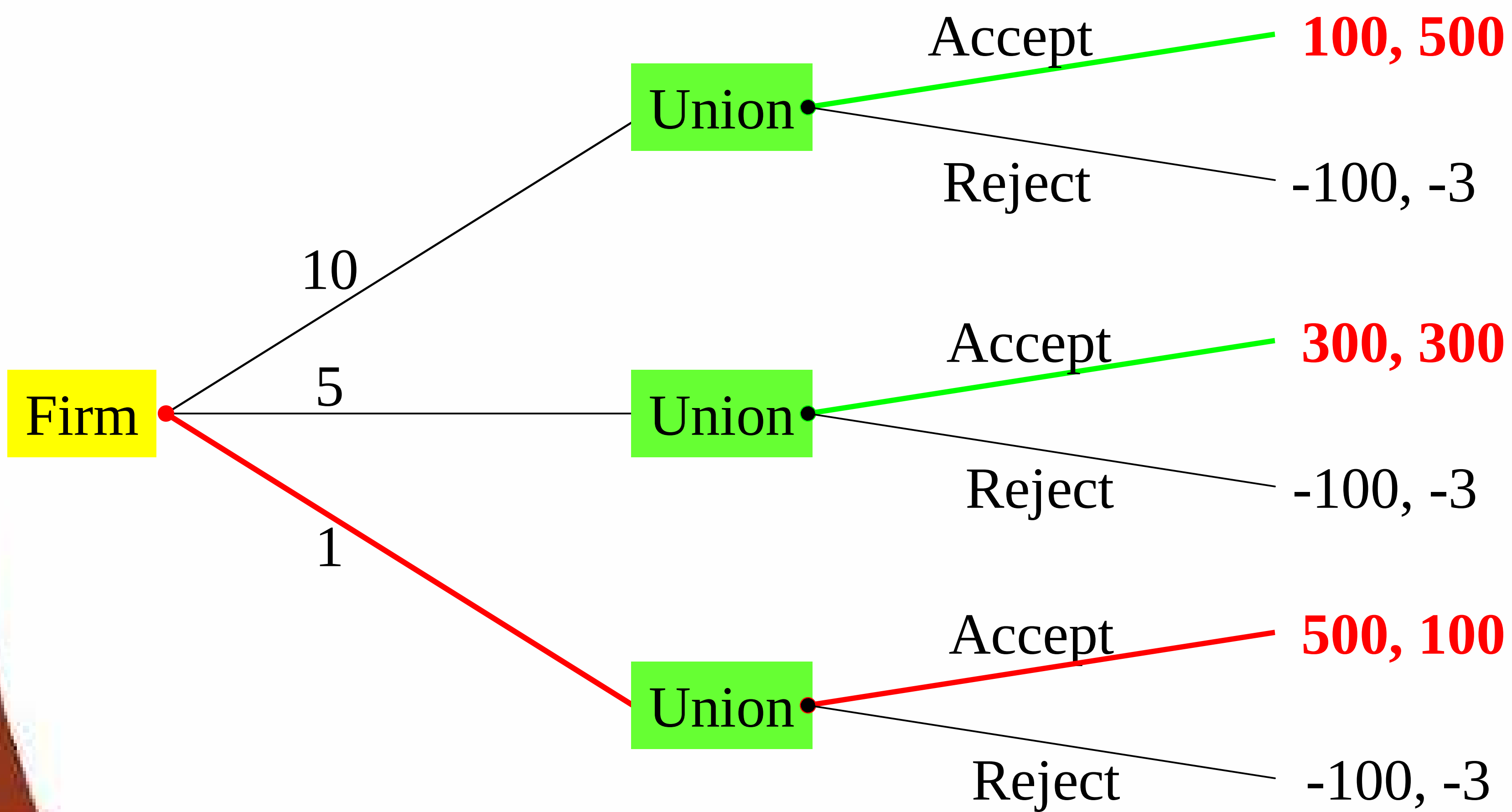
To Summarize:

- We have identified many combinations of Nash equilibrium strategies.
- In all but one the union does something that isn't in its self interest (and thus entail threats that are not credible).
- Graphically:

There are 3 Nash Equilibrium Outcomes!



Only 1 Subgame-Perfect Nash Equilibrium Outcome!



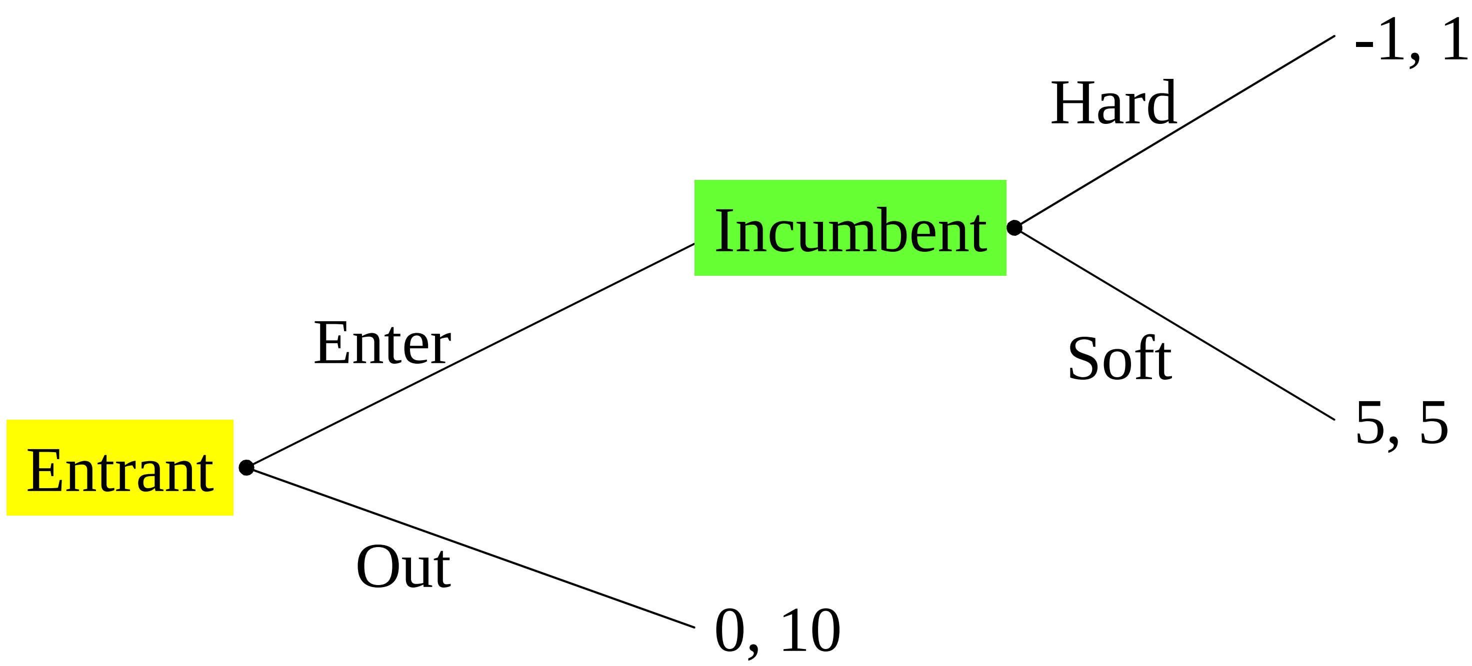
Bargaining Re-Cap

- In take-it-or-leave-it bargaining, there is a first-mover advantage.
- Management can gain by making a take-it-or-leave-it offer to the union. But...
- Management should be careful; real world evidence suggests that people sometimes reject offers on the the basis of "principle" instead of cash considerations.

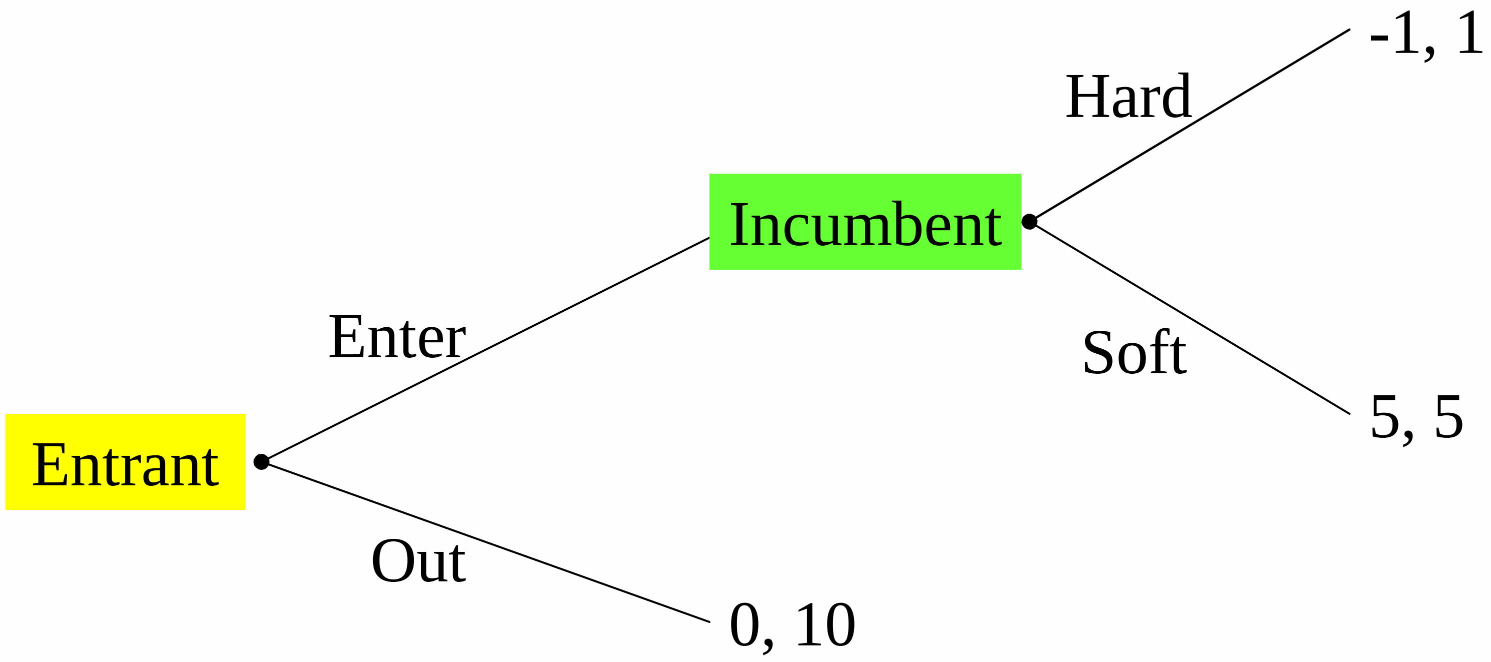
Pricing to Prevent Entry: An Application of Game Theory

- Two firms: an incumbent and potential entrant.
- Potential entrant's strategies:
  - Enter.
  - Stay Out.
- Incumbent's strategies:
  - {if enter, play hard}.
  - {if enter, play soft}.
  - {if stay out, play hard}.
  - {if stay out, play soft}.
- Move Sequence:
  - Entrant moves first. Incumbent observes entrant's action and selects an action.

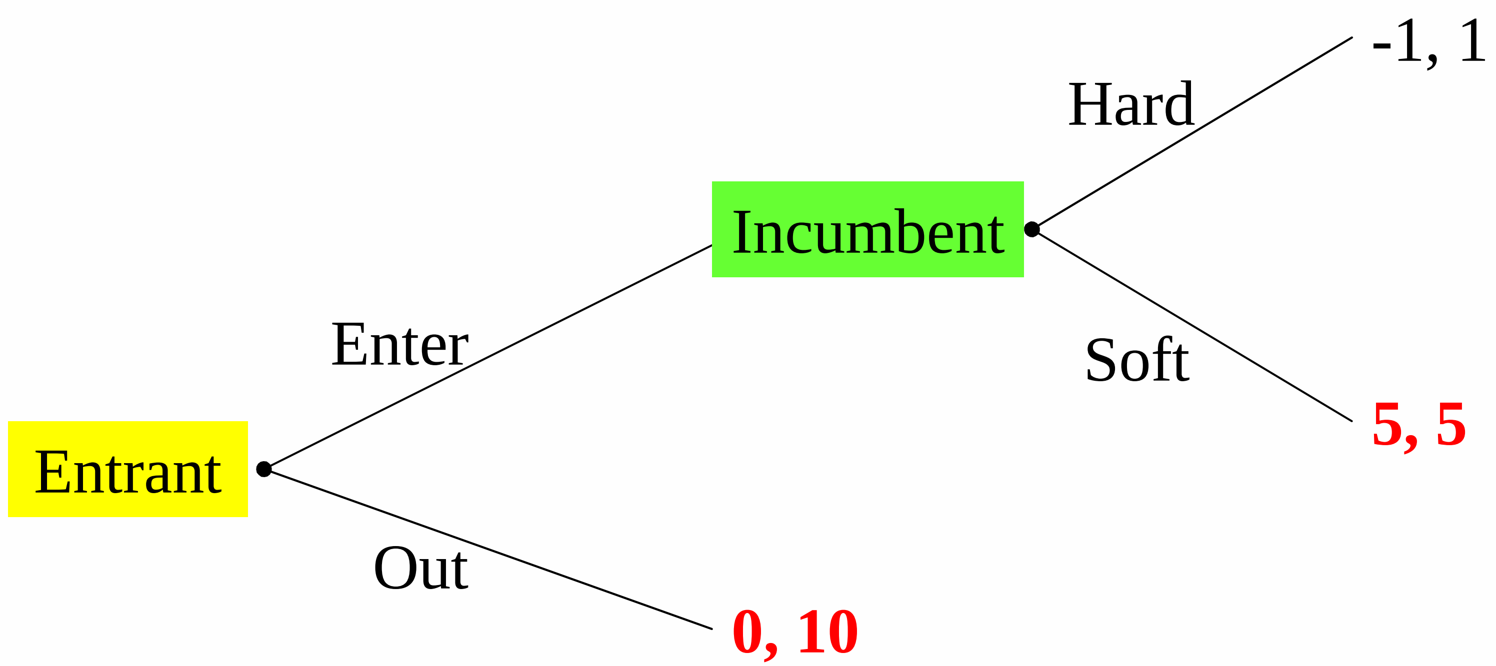
The Pricing to Prevent Entry Game in Extensive Form



Identify Nash and Subgame Perfect Equilibria

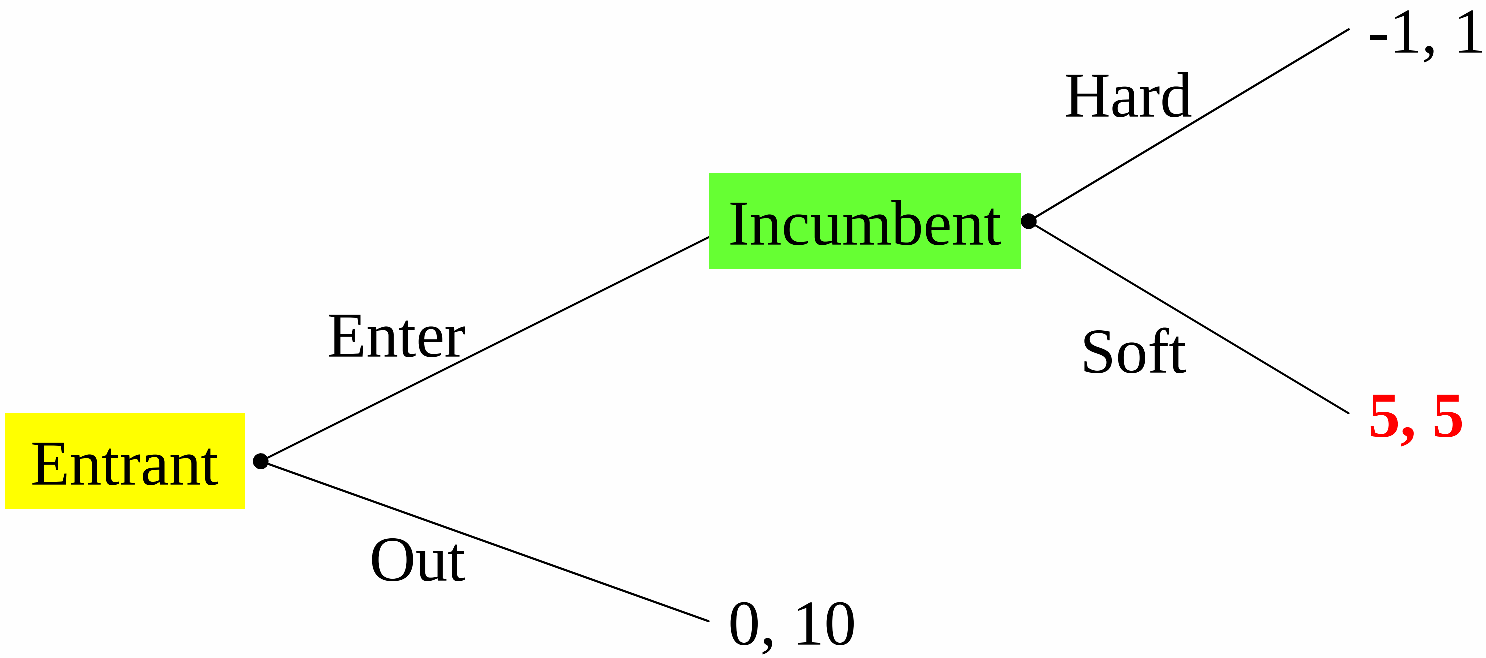


Two Nash Equilibria



Nash Equilibria Strategies {player 1; player 2}:
{enter; If enter, play soft}
{stay out; If enter, play hard}

One Subgame Perfect Equilibrium



Subgame Perfect Equilibrium Strategy:
{enter; If enter, play soft}

Insights

- Establishing a reputation for being unkind to entrants can enhance long-term profits.
- It is costly to do so in the short-term, so much so that it isn't optimal to do so in a one-shot game.