A News Publication of the Decision Sciences Institute

DECISION LINE

Vol. 28, No. 3 May 1997

An Update on the San Diego Program

■ by David Olson, Texas A&M University

We are currently reviewing over 800 paper submissions for the 1997 Decision Sciences Institute Annual Meeting. The Instructional Innovation Award Competition and the Elwood S. Buffa Doctoral Dissertation Competition are both underway as well. Theme chairs are organizing sessions related to international, environmental, health care, public sector, and interdisciplinary business. The New Faculty and Doctoral Student Consortium plans are underway to offer participants useful and timely presentations.

We have a number of case activities in progress. Robert Carraway is conducting reviews for the Case Competition. The deadline for submissions to that competition

See **PROGRAM CHAIR**, page 34

PRESIDENT'S LETTER

■ James R. Evans, University of Cincinnati

I don't have a degree in business; all my academic degrees come from industrial engineering programs. However, I was fortunate to enter the job market in the mid-70s when many business schools were developing strengths in quantitative disciplines and hired this expertise from engineering. As such, I did not have the opportunity to learn about the Decision Sciences Institute as a graduate student.

I joined the Institute and began attending annual meetings a few years after joining the faculty at the University of Cincinnati, primarily because the meetings were being held in some really neat places like San Francisco, New Orleans, and Las Vegas. (People still think I'm kidding about this.) However, I soon learned that DSI had much to

See **PRESIDENT'S LETTER**, page 46

Inside This Issue

FEATURES

SPECIAL REPORTS

DEPARTMENTS

http://www.gsu.edu/~dsiadm/

■ Barbara B. Flynn, Babcock Graduate School of Management, Wake Forest University

The Art and Science of Mathematical Sin

by H. Arsham, University of Baltimore

Our culture has always reflected a lack of comfort with the notion of zero. Witness humor such as "two plus zero still equals two, even for large values," and popular cultural retorts of similar tone. A similar uneasiness exists regarding infinity, whose proper use first rests on a careful definition of what is finite. Are we mortals hesitant to admit to our finite nature? Such commentary reflects an underlying awkwardness in manipulation equations where the notions of zero and infinity present itself. It is not simply a problem of ignorance by young novitiates. The same errors are commonly committed by seasoned practitioners. Nay, even educators.

It should not then come as a total surprise when reading journals, prestigious or otherwise, to find authors committing the sin of dividing by zero and mishandling the concept of infinity. These errors are found in simple mathematical treatments and in complex operations such as performing column ration tests in a simplex tableau. I recall one such instance where the author reached the stated conclusion that 2/0 = Infinity. Typographical error? Confusion? Willful sin?

Dividing by zero is a mathematical sin! If we persist in retaining such errata in our professional publications, an unwitting or unscrupulous person could utilize the result to subsequently show that 1=2!, as follows:

$$a^2 - a^2 = a^2 - a^2 \text{ for any finite } a,$$

$$a(a - a) = (a - a)(a + a),$$

dividing both sides by $(a - a)$ gives

$$a = 2a,$$

now, dividing by a gives

$$1 = 2.$$

Voila!

This result follows directly from the conclusion that it is a legal operation to divide by zero! If you divide 2 by zero even on a simple, inexpensive calculator, the calculator will indicate an error condition. Could this cheap calculator know something that we practitioners do not? Viewing this issue from the perspective of limits, when considering Lim $(2/a)$ as a approaches zero (not equal to zero), neither the left nor right limit exists. In other words, if one divides 2 by a very small positive number close to zero, the result is a very large positive number while dividing 2 by a very small negative number close to zero produces a very large negative number. Since the two results are not equal, therefore the limit does not exist. Neither does the limit of each side exist.

It is not a simple question of chastising one author or one publisher or one student. Unfortunately what I find is that this is not at all an uncommon practice. Many references in OR can be found committing this error. And if educators profess division by zero as an appropriate mathematical practice, we should not be surprised to see this error persist among students just as some authors themselves learned this abysmal practice from their own teachers. These occurrences are a painful, vivid demonstration of a widespread misconception.

The notion of zero we have was introduced in the Middle Ages by Arabian scholars as a superior mathematical construction compared with the then prevalent Roman numerals, which do not contain the notion of zero. When these scholarly treatises were being translated by European accountants, they translated 1, 2, 3, ... and upon reaching zero, pronounced, "empty." Nothing! The scribe asked what to write and was instructed to draw an empty hole, thus introducing the present notation for zero. It may be considered frivolous hyperbole to suggest that the demise of the Roman Empire was due to the absence of zero in their number system, but one can only ponder the fate of our civilization given the difficulty our culture seems to have with the presence of zero in our number system. Natural numbers are real numbers. One car, two trees . . . . What about negative numbers? The negative sign is an extension of the number system used to indicate directionality. Sacrilegious as it may sound on first impression, the notation of zero is at heart nothing more than a directional separator. It is in actuality, "nothing." A numerical value (other than zero) divided into "nothing" inherently results in nothing. This is not a simple calculation exercise. Rather it reaches to the nature of the underlying physical reality.

Another common error is often found in textbooks that announce the finding that the square root of 4 is ±2. When this writer confronted an author guilty of this practice, observing that one number cannot be equal to two different numbers, the reply received was, "Check it for yourself by squaring both sides." This writer advised that following his argument one could also demonstrate that one is equal to minus one. An observer witnessing this exchange jumped in, volunteering the results of the computation performed with a calculator as producing a single result of plus 2, declaring "He's right." Solving the equation $x^2=4$ has two solutions, $x=\pm\sqrt{4}$. The square root of 4 is 2, therefore $x=\pm2$. This correct result is distorted when one goes on to write $x=\sqrt{4}$ and concludes that this latter result is ±2. This is the genealogy of this error. There is a clear distinction here and an important difference which the careful reader will note.

Our conclusion is that these two errant views are widely held among practitioners of decision sciences and, unsurprisingly, by their students. Sadly, these persistent errors do not exist in isolation in a classroom or academic text. Important conclusions are inappropriately drawn after a witting, or unwitting, division by zero, leading the calculator to subsequently conclude, "therefore . . ." as he or she goes on to some consequent insight. This writer uses the 1=2 and $\sqrt{4}$ examples as experiments in every one of his classes. Inevitably, almost half of the class responds incorrectly. We would suggest readers who teach try a similar experiment in their classes. Go forth, and sin no more! ■