OR newsletter

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No facts, please. We're government!

H M Government, which routinely compels local authorities and other bodies to publish detailed analyses of their expenditure, fails to do the same in respect of its own much greater spending. Can OR do anything about this? Page 20

Merry Christmas
Happy New Year

-1 = 1 = 2, right?

by Dr H Arsham, University of Baltimore

Imprecise mathematical thinking is by no means unknown in OR. But, argues H Arsham, we need to think more clearly if we are to keep out of trouble

Our culture has always reflected a lack of comfort with the notion of zero. Witness humour such as "Two plus zero still equals two, even for large values", and popular cultural retorts of similar tone. A similar uneasiness exists regarding infinity. Such lighthearted comments are a reflection of an underlying awkwardness in manipulating equations where the notion of zero presents itself. The problem is hardly limited to young students grappling with an idea which has often been mangled. It can frequently be found as well in prestigious texts published by mainstream publishers.

Dividing by zero is a sin!

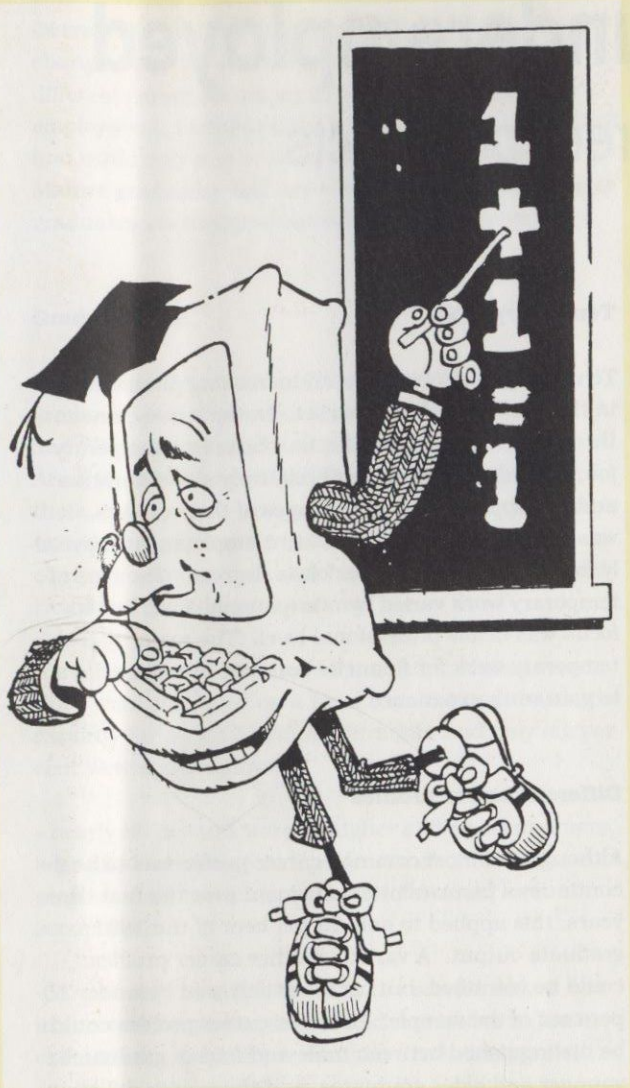
Reading the fifth edition of a book (Introduction to Management Science, by B. W. Taylor III, Prentice Hall, New Jersey, 1996), I found the author dividing 2 by zero in a Simplex tableau performing a column ratio test, with the stated conclusion, 2 / 0 = infinity. A silly typographical error? Confusion? Wilful sin? A telephone call bringing the obvious error to the attention of the publisher for correction in future editions was met with an astonishing return call from the editor of the text still insisting that the result of 2 / 0 *was* infinity! Although both the author and editor insist on this computational outcome, they nonetheless somehow decline to continue the Simplex calculation based on this result, contrary to the logic of their conclusion.

Dividing by zero can get you into trouble!

Dividing by zero is a mathematical sin! If we persist in retaining such errata in our educational texts, an unwitting or unscrupulous person could utilise the result to subsequently show that 1 = 2, as follows:

$a^2 - a^2 = a^2 - a^2$ for any finite a
$a(a-a) = (a-a)(a+a)$. Dividing both sides by (a-a) gives
$a = 2a$ Now, dividing by a gives
$1 = 2$ Voila!

This result follows directly from the conclusion that it is a legal operation to divide by zero! With the editor still unconvinced, in discussing the issue this writer suggested if you divide 2 by zero even on a simple, inexpensive calculator, the calculator will indicate an error condition. Could this cheap calculator know something the editor does not? Viewing this issue from the perspective of limits, when considering Lim (2/a) as a approaches zero (not equal to zero), neither the left nor right limit exists. In other words, if one divides 2 by a very small positive number close to zero, the result is a very large positive number while dividing 2 by a very small negative number close to zero produces a very large negative number. Since the two results are not equal, therefore the limit does not exist. Neither does the limit of each side exist. Since the publisher professes "mathematically rigorous techniques" for the text in their management catalogue for 1996 and basks in the claim of being a "widely adopted" text, it is all the more important to bring this abysmal error of dividing by zero to the attention of the OR community. This is not a simple question of chastising one author or one publisher. Unfortunately I find that this is not at all an uncommon practice. And if an educator professes division by zero as an appropriate mathematical practice, we should not be surprised to see this error persist among his students just as the author himself learned this abysmal practice from his own teachers. The author lists over 20 educators as reviewers, including the editor. This is a painful, vivid demonstration of how widespread this misconception is. It should also be noted that this particular text and author are cited only to illustrate a widespread problem. By every appearance, the text otherwise has much to recommend it, both for its scholarship, application, and readability.



The notion of zero was introduced in the Middle Ages by Arabian scholars as a superior mathematical construction compared with the then prevalent Roman numerals which did not contain the notion of zero. When these scholarly treatises were being translated by European accountants, they translated 1, 2, 3, ... and upon reaching zero, pronounced, "empty." Nothing! The scribe asked what to write and was instructed to draw an empty hole, thus introducing the present notation for zero. It may be considered frivolous hyperbole to suggest that the demise of the Roman Empire was due to the absence of zero in their number system, but one can only ponder the fate of our civilisation given the difficulty our culture seems to have with the presence of zero in our number system.

Natural numbers are real numbers. One car, two trees... What about negative numbers? The negative sign is an extension of the number system used to indicate directionality. Sacrilegious as it may sound on first impression, the notation of zero is at heart nothing more than a directional separator. It is in actuality, "nothing." A numerical value (other than zero) divided into "nothing" inherently results in nothing. This is not a simple calculation exercise. Rather it reaches to the nature of the underlying physical reality.

How to prove that 1 = -1

Another common error is found in other textbooks which announce the finding that the square root of 4 is ± 2. When this writer confronted an author guilty of this practice observing that one number cannot be equal to two different numbers, the reply received was "check it for yourself by squaring both sides." He followed with self-satisfaction, "you see!". This writer advised that following his argument one could also demonstrate that one is equal to minus one. An observer witnessing this exchange jumped in volunteering the results of the computation performed with a calculator as producing a single result of plus 2 declaring "he is right." Solving the equation $x^2 = 4$ has two solutions, $x = \pm\sqrt{4}$. The square root of 4 is two, therefore $x = \pm 2$. This correct result is distorted when one goes on to write $x = \sqrt{4}$ and concludes that this latter result is ± 2. This is the genealogy of this error. There is a clear distinction here and an important difference which the careful reader will note.

Errant views

Our conclusion is that these two errant views are widely held among authors of OR texts and unsurprisingly, by their students. Sadly, these persistent errors do not exist in isolation in a classroom or academic text. Important conclusions are inappropriately drawn after a witting or unwitting division by zero, leading the calculator to subsequently conclude, "therefore...." as he or she goes on to some consequent insight. This writer uses the 1 = 2 and $\sqrt{4}$ examples as experiments in every one of his classes. Inevitably, almost half of the class responds incorrectly. We would suggest readers who teach try a similar experiment in their classes.

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