**Multi-Means Comparisons: Analysis of Variance (ANOVA)**

The tests we have learned up to this point allow us to test hypotheses that examine the difference between only two means. Analysis of Variance or ANOVA will allow us to test the difference between two or more means. ANOVA does this by examining the ratio of variability between two conditions and variability within each condition. For example, say we give a drug that we believe will improve memory to a group of people and give a placebo to another group of people. We might measure memory performance by the number of words recalled from a list we ask everyone to memorize. A t-test would compare the likelihood of observing the difference in the mean number of words recalled for each group. An ANOVA test, on the other hand, would compare the variability that we observe between the two conditions to the variability observed within each condition. Recall that we measure variability as the sum of the difference of each score from the mean. When we actually calculate an ANOVA we will use a short-cut formula

Thus, when the variability that we predict between the two groups is much greater than the variability we don't predict within each group, then we will conclude that our treatments produce different results.

**An Illustrative Numerical Example for ANOVA**

Consider the following (small integers, indeed for illustration while saving space) random samples from three different populations.

With the null hypothesis:
H0: µ1 = µ2 = µ3,
and the alternative:
Ha: at least two of the means are not equal.

At the significance level = 0.05, the critical value from F-table is
F 0.05, 2, 12 = 3.89.

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|   | **Sum** | **Mean** |
| **Sample P1** | 2 | 3 | 1 | 3 | 1 | 10 | 2 |
| **Sample P2** | 3 | 4 | 3 | 5 | 0 | 15 | 3 |
| **Sample P3** | 5 | 5 | 5 | 3 | 2 | 20 | 4 |

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Demonstrate that, SST=SSB+SSW.
That is, the sum of squares total (SST) equals sum of squares between (SSB) the groups plus sum of squares within (SSW) the groups.

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**Computation of sample SST:** With the grand mean = 3, first, start with taking the difference between each observation and the grand mean, and then square it for each data point.

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|   | **Sum** |
| **Sample P1** | 1 | 0 | 4 | 0 | 4 | 9 |
| **Sample P2** | 0 | 1 | 0 | 4 | 9 | 14 |
| **Sample P3** | 4 | 4 | 4 | 0 | 1 | 13 |

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Therefore SST = 36 with d.f = (n-1) = 15-1 = 14

**Computation of sample SSB:**

Second, let all the data in each sample have the same value as the mean in that sample. This removes any variation WITHIN. Compute SS differences from the grand mean.

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|   | **Sum** |
| **Sample P1** | 1 | 1 | 1 | 1 | 1 | 5 |
| **Sample P2** | 0 | 0 | 0 | 0 | 0 | 0 |
| **Sample P3** | 1 | 1 | 1 | 1 | 1 | 5 |

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Therefore SSB = 10, with d.f = (m-1)= 3-1 = 2 for m=3 groups.

**Computation of sample SSW:**

Third, compute the SS difference within each sample using their own sample means. This provides SS deviation WITHIN all samples.

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|   | **Sum** |
| **Sample P1** | 0 | 1 | 1 | 1 | 1 | 4 |
| **Sample P2** | 0 | 1 | 0 | 4 | 9 | 14 |
| **Sample P3** | 1 | 1 | 1 | 1 | 4 | 8 |

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SSW = 26 with d.f = 3(5-1) = 12. That is, 3 groups times (5 observations in each -1)

Results are: SST = SSB + SSW, and d.fSST = d.fSSB + d.fSSW, as expected.

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Now, construct the ANOVA table for this numerical example by plugging the results of your computation in the ANOVA Table. Note that, the Mean Squares are the Sum of squares divided by their Degrees of Freedom. F-statistics is the ratio of the two Mean Squares.

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| **The ANOVA Table** |
| Sources of Variation | Sum of Squares | Degrees of Freedom | Mean Squares | F-Statistic |
| Between Samples | 10 | 2 | 5 | 2.30 |
| Within Samples | 26 | 12 | 2.17 |   |
| Total | 36 | 14 |   |   |

Conclusion: There is not enough evidence to reject the null hypothesis H0.

**The Logic behind ANOVA:** First, let us try to explain the logic and then illustrate it with a simple example. In performing the ANOVA test, we are trying to determine if a certain number of population means are equal. To do that, we measure the difference of the sample means and compare that to the variability within the sample observations. That is why the test statistic is the ratio of the between-sample variation (MSB) and the within-sample variation (MSW). If this ratio is close to 1, there is evidence that the population means are equal.

Here is a good application for you: Many people believe that men get paid more in the business world, in a specific profession at specific level, than women, simply because they are male. To justify or reject such a claim, you could look at the variation within each group (one group being women's salaries and the other group being men's salaries) and compare that to the variation between the means of randomly selected samples of each population. If the variation in the women's salaries is much larger than the variation between the men's and women's mean salaries, one could say that because the variation is so large within the women's group that this may not be a gender-related problem.

Now, getting back to our numerical example of the drug treatment to improve memory vs. the placebo, we notice that: given the test conclusion and the ANOVA test's conditions, we may conclude that these three populations are in fact the same population. Therefore, the ANOVA technique could be used as a measuring tool and statistical routine for quality control as described below using our numerical example.

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**Construction of the Control Chart for the Sample Means:** Under the null hypothesis, the ANOVA concludes that µ1 = µ2 = µ3; that is, we have a "hypothetical parent population." The question is, what is its variance? The estimated variance (i.e., the total mean squares) is 36 / 14 = 2.57. Thus, estimated standard deviation is = 1.60 and estimated standard deviation for the means is 1.6 / 5½  0.71. Under the conditions of ANOVA, we can construct a control chart with the warning limits = 3 ± 2(0.71); the action limits = 3 ± 3(0.71). The following figure depicts the control chart.

**Conditions for Using ANOVA Test:** The following conditions must be tested prior to using ANOVA test otherwise the results are not valid: Randomness of the samples, Normality of populations, and [Equality of Variances](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/BartletTest.htm%22%20%5Ct%20%22new) for all populations.

**You May Ask Why Not Using Pair-wise T-test Instead ANOVA?** Here are two reasons: Performing pair-wise t-test for K populations, you will need to perform,

K(K-1)/2 pair-wise t-test. Now suppose the significance level for each test is set at 5% level, then the overall significance level would be approximately equal to

0.05K(K-1)/2. For example, for K = 5 populations, you have to performing 10 pair-wise t-tests, moreover, the overall significance level is equal to 50%, which is too high type-I error for any statistical decision making.

You might like to use [ANOVA: Testing Equality of Means](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/anova.htm%22%20%5Ct%20%22new) for your computations, and then to interpret the results in managerial (not technical) terms.



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