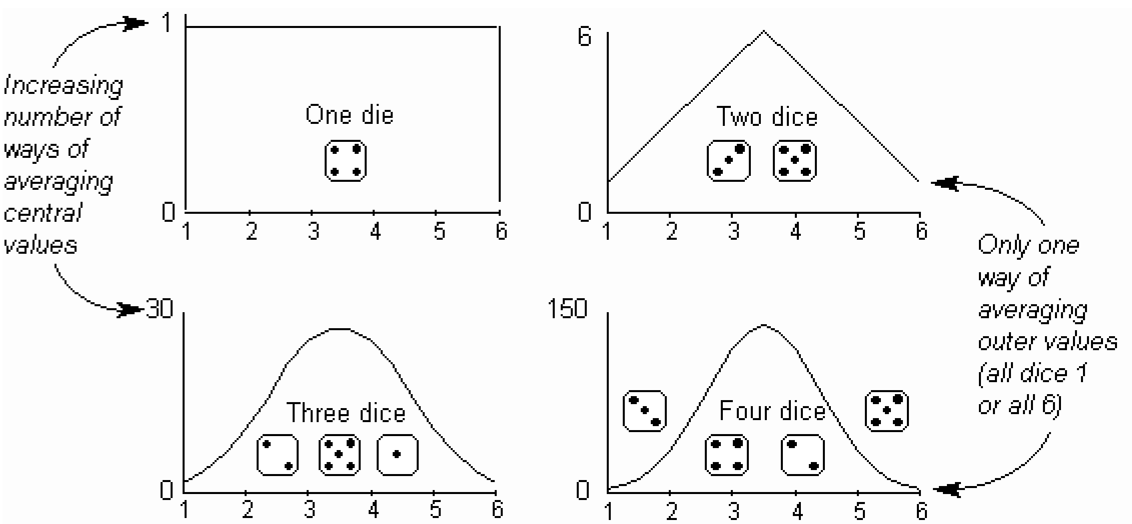
Unifications	of Techniques: See the Wholeness and Many-Fold ness
Population Continuous R.V.	μ, σ^2 Expected value and Variance both unknown Must be estimated with confidence. Homogenous population
Population Discrete R.V.	$\mu = \sum(xi\ pi) = E(x)$; $\sigma^2 = (\mathrm{E}x^2) - (Ex)^2 = \sum(xi^2\ pi) - [\sum(xi\ pi)]^2$; $\sigma = \sqrt{\sigma^2}$
Sample (s)	$\overline{x} = \frac{\sum x}{n}$; $s^2 = \frac{\sum(x-\overline{x})^2}{n-1} = \frac{SS}{n-1}$, $ss = Sum(x^2) - (Sum\ x)^2/$ n, C.V. $= \frac{s}{\overline{x}}$
Probability:	P(A given B)=P(A|B) = P(A and B)/ P (B) = $\frac{P(A \cap B)}{P(B)}$ P(A or B) = P(A∪B) = P(A) + P(B) - $P(A \cap B)$ A and B are independent if P(A|B) = P(A)

Binomial *Probability Distribution*	**x** success in **n** trial with $\boldsymbol{\pi}$ is probability success P(x) = $\frac{n!}{x!\,(n-x)!}\pi^x (1-\pi)^{n-x}$ μ =n π; σ^2 = n $\pi(1-\pi)$

Central limit theorem: Sampling distribution of mean tends to be normal density as the (fixed) sample size increases. μ=$\mu_{\overline{x}}$ and $\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$ if

σ *unknow, use* $\boldsymbol{s}$

CLT in Action:



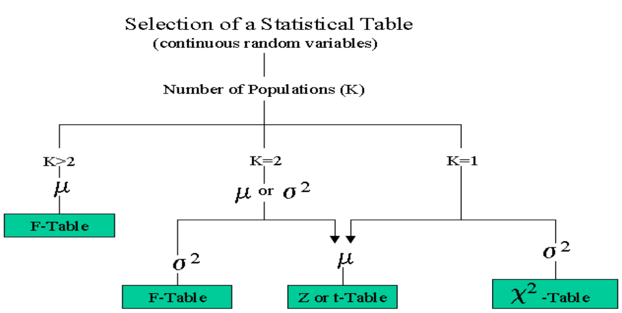
For the parent population the **expected value** is:

μ =$\sum(xi\ pi) = E(x)$ = 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6) = 21/6 which the same for the other 3 sampling means distributions.

For the parent population the **variance** is:

$\sigma^2 = (\mathrm{E}x^2) - (Ex)^2 = \sum(xi^2\ pi) - (Ex)^2$ = 1(1/6) + 4(1/6) + 9(1/6) + 16(1/6) + 25(1/6) + 36(1/6) – $(21/6)^2$ = 4.04

The variance for the others is $\frac{\sigma^2}{n}$ = 4.04/2= 2.02, 4.04/3 = 1.37, 4.04/4 = 1.01, respectively, it get smaller as sample size increases.



	For Population	For sampling distribution with sample size = n
Z-statistic	Z=$\frac{x-\mu}{\sigma}$ to make N(0,1)	$Z_{\bar{x}}$=$\frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}}}$ = $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ ~= $\frac{\bar{x}-\mu}{s/\sqrt{n}}$ (in that $\mu_{\bar{x}} = \mu$ *and* $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ and large n, invoke the CLT)
T-statistic	Normal	$T_{\bar{x}}$= $\frac{\bar{x}-\mu}{s/\sqrt{n}}$ with small n, but population is Normal

Notice the difference: The Z-score, Z-transformation = $\frac{x-\bar{x}}{s}$, is used to make data dimensionless, often for comparison purposes. For example price of the houses and their sized in Washington DC., and Tokyo.

Estimation with Confidence

	1 Population	2 Populations
Population mean μ	$\bar{x} \pm z_{\frac{\alpha}{2}}\, \sigma_{\bar{x}} = \bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ $\bar{x} \pm t_{\frac{\alpha}{2}}\, \sigma_{\bar{x}} = \bar{x} \pm t_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ (with v = n-1)	$\overline{x_1}$ - $\overline{x_2} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ $\overline{x_1}$ - $\overline{x_2} \pm t_{\frac{\alpha}{2}} \sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$ (With pooled estimate for S: s_p^2 =$\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$) (v = $n_1 + n_2 - 2$)
Population proportion (probability) (p=$\frac{x}{n}$) Sample are sufficiently large	μ_p = π true population proportion of success. $\sigma_p = \sqrt{\frac{\pi\,(1-\pi)}{n}}$ P $\pm z_{\frac{\alpha}{2}} \sqrt{\frac{p\,(1-p)}{n}}$	P_1 - $P_2 \pm z_{\frac{\alpha}{2}}\, \sigma_{(p1-p2)}$ = P_1 - $P_2 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p1\,(1-p1)}{n1} - \frac{p2\,(1-p2)}{n2}}$
Means: 2pops Matched pairs	Known also as the "*before-and-after*" test Large sample size (CLT) if the d is not normal	$\mu_d = (\mu_1 - \mu_2)$ $\bar{d} \pm z_{\frac{\alpha}{2}} \frac{\sigma_d}{\sqrt{n}} = \bar{d} \pm z_{\frac{\alpha}{2}} \frac{S_d}{\sqrt{n}}$ (if σ_d *unknown with large sample*) $\bar{d} \pm tz_{\frac{\alpha}{2}} \frac{\sigma_d}{\sqrt{n}} = \bar{d} \pm t_{\frac{\alpha}{2}} \frac{S_d}{\sqrt{n}}$ (if σ_d *unknown with small sample*
Variance σ^2	$\frac{(n-1)s^2}{X^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)s^2}{X^2_{(1-\frac{\alpha}{2})}}$	$\frac{s_1^2}{s_2^2\, F(n2-1,n1-1,\frac{\alpha}{2})} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{s_1^2 F(n1-1,n2-1,\frac{\alpha}{2})}{s_2^2}$
Determination of sample size for Continuous and Discrete R.V.	with d margin of error: n = $\frac{z^2_{\alpha/2}\,\sigma^2}{d^2}$ With d margin of error for proportion: n = $\frac{z^2_{\alpha/2}\;\pi\,(1-\pi)}{d^2}$, *you may use* $\pi = 0.5$	

Hypothesis Testing

	1Population	2Populations
	1tailed: Ha: > or (<) Rejection region: $Z > z_{\alpha}$ or ($Z < - z_{\alpha}$)	**2tailed: Ha: ≠** Rejection region: $Z > z_{\frac{\alpha}{2}}$ or $Z < - z_{\frac{\alpha}{2}}$
Pop mean μ	Ho: μ= μo Ha: μ≠ μo For one-sided use $Z > z_{\frac{\alpha}{2}}$ or $Z < - z_{\frac{\alpha}{2}}$) $Z= \frac{\bar{x}- \mu o}{\sigma_{\bar{x}}} \sim = \frac{\bar{x}- \mu o}{s/\sqrt{n}}$ Large sample size to invoke CLT.	Ho: μ1- μ2 = 0 Ha: μ1- μ2 ≠ 0 $Z = \frac{\overline{x1}- \overline{x2}}{\sigma_{\overline{x1}} - \sigma_{\overline{x2}}} \sim = \frac{\overline{x1}- \overline{x2}}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$ If variances are almost the same, then use $t = \frac{\overline{x1}- \overline{x2}}{\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ with table v = n1+n2-2
Population proportion (probability) ($\pi \rightarrow p=\frac{x}{n}$) Sample are sufficiently large	Ho: $\pi = \pi o$ Ha: $\pi \neq \pi o$ $Z = \frac{P - \pi o}{\sqrt{\frac{\pi o\,(1-\pi o)}{n}}}$ Large sample size to invoke CLT.	Ho: $\pi 1 - \pi 2 = 0$ Ha: $\pi 1 - \pi 2 \neq 0$ $Z= \frac{P1 - P2}{\sigma of_{(P1-P2)}} = \frac{P1 - P2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ With $p=\frac{x1+x2}{n1+n2}$ and q = 1-p, large sample size.
Population variance	Ho: $\sigma^2 = \sigma_0^2$ Ha: $\sigma^2 \neq \sigma_0^2$ 2tailed: $X^2 > X^2_{1-\alpha/2}$ or $X^2 < X^2_{\alpha/2}$ 1tailed: $X^2 > X^2_{1-\alpha}$ (or $X^2 < X^2_{\alpha}$) With v = n-1 $X^2 = \frac{(n-1)s^2}{\sigma_0^2}$ Population is normal	Ho: $\frac{\sigma_1^2}{\sigma_2^2} = 1$ **1tailed:** Ha: $\frac{\sigma_1^2}{\sigma_2^2} > 1$ F= $\frac{s_1^2}{s_2^2}$, critical value F_{α} (n1-1, n2-1) **1tailed:** Ha: $\frac{\sigma_1^2}{\sigma_2^2} < 1$ F= $\frac{s_1^2}{s_2^2}$, critical value $1/F_{\alpha}$ (n2-1, n1-1) **2tailed:** Ha: $\frac{\sigma_1^2}{\sigma_2^2} \neq 1$, **F** = $\frac{Larger\ variance}{Smaller\ variance}$, critical value $F_{\alpha/2}$ (n1-1, n2-1), Populations are normal
Population mean for Pair matched	There is dependency, known also as the "*before-and-after*" test. Large sample size (CLT) if the d is not normal	Ho: μ1- μ2 = 0 Ha: μ1- μ2 ≠ 0, **Z** = $\frac{\bar{d}}{\sigma_d/\sqrt{n}} = \frac{\bar{d}}{S_d/\sqrt{n}}$