#### Numerical Examples for Statistical Tables

The presentation of the statistical tables is not universal. Some statistical textbooks authors’ enjoy given tabular values of the right-tail probabilities, while for others left-tail probabilities are preferred. Even within each of these groups you will find some differences in presenting each table differently than others, never in a unified format. This lack of uniformity often confuses most of students while learning statistics.

The following presents some numerical examples of common statistical tables with some applications. You may like using [The P-values for the Popular Distributions](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/pvalues.htm%22%20%5Ct%20%22new) JavaScript.

**[Binomial Probability](http://home.ubalt.edu/ntsbarsh/Business-stat/opre504.htm%22%20%5Cl%20%22rBinomial%22%20%5Ct%20%22new)**

X B(n, p), read, the random variable X has a binomial distribution with parameters n trials, and probability of a success is p.

**Example:** Find probability of at most k = 3 success from B(n = 7, p = 0.4). Using any Binomial table, one should get:
P[k  3] = 0.7102.
Using [The P-values for the Popular Distributions](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/pvalues.htm%22%20%5Ct%20%22new) JavaScript, one gets:

P[k  3] = 1 – P[k  4] = 1 – 0.2898 = 0.7102.

Questions for you: Which of the following two events is more likely to happen? Getting exactly 6 heads in tossing a fair coin (i.e, p=1/2), n = 10 times or tossing it n=20 times. Why?

**Application:** A traveling salesman has find that the probability of a sale on a single contact is 0.02. If the salesman contacts 200 prospects, find the probability that he will make at least one sale.

P[at least one sale] = 1 – P[no sale] = 1 – (1-0.02)200 = 1 – (0.98)200 = 98%

**[Normal Density Function](http://home.ubalt.edu/ntsbarsh/Business-stat/opre504.htm%22%20%5Cl%20%22rnd%22%20%5Ct%20%22new)**

X N(0, 1), read, the random variable X is distributed Normally with mean, and variance 0, and 1, respectively.

A Fact: If X N(), then

 = (X N(0, 1)

**Example:** Let X N(1, 2), compute P(X  5.21)
P[ (X 1) / 2  (5.21 1) / 2] = P(Z  2.105)  P(Z  2.11) = .4826 + .5 = .9826
Notice that P(Z  0) = .5

Similarly, P(X  2.1) = P(Z  (2.11) / (2)) = P(Z  .55) = 0.5 - .2088 = .2912
Using [The P-values for the Popular Distributions](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/pvalues.htm%22%20%5Ct%20%22new) JavaScript, the 2p-value is:

P[| Z |  2.1] = 0.582.

Questions for you: Compute P( X  3),  P(1  X  4),  P(X 1),   find the value of  such that P(X ) = 0.4515

**Applications:**

1. Testing hypotheses on the population’s mean, with a known variance, at a given significance level .

H0: 0     Ha: 0

A Fact: Given X  N(**?**, ) and having a random realization of size n: x1, x 2, ..., xn, then

Z = [xbarn - ] / ( / n1/2)  N(0,1).

Notice that in most cases, the standard deviation ( is unknown. However, one may use the sampling estimate S for  provided the sample size is large enough, say, over 30.)

Given n = 4, xbar4 = 492  , test H0:  at significance level  = 0.05 if  = 16?
The Z-statistic is Z = [492  500] / [16 / (41/2)] = -1, however the tabulated critical Z-value is Z.025 = 1.96
Conclusion: No reason to reject H0.

Question for you: Given the same sampling information, test H0:  = 505 vs. Ha:  505.

2. Setting a confidence interval on the mean, variance known.
Given xbar4 = 492   construct a 95% confidence interval for  given  = 16

P [xbar - Z  / n1/2    xbar + Z  / n1/2]  1- 

Plugging in the numerical values, one gets:
P[476.3    507.7]  0.95

Notice the **Duality** between the test of hypothesis and confidence interval.

Question for you:   Given the same sampling information, construct a 90% confidence interval for  given the same information.

3. Central Limit Theorem (CLT)

A Fact: If E(X) = , Var(X) = , then

(xbar ) / ( / n1/2) N(0,1),

 for large n, say, n  30

As a strong result, the CLT implies that if the sample size is large enough, then one may relax the normality condition whenever dealing with the question of testing or constructing confidence interval for population’s mean ().

**[T-Density Function](http://home.ubalt.edu/ntsbarsh/Business-stat/opre504.htm%22%20%5Cl%20%22rtdistributions%22%20%5Ct%20%22new)**

A Fact: If X N(, ?), then

[xbar ] / [S / n 1/2]  t n-1

**Example:** Find t such that P(T11 > t) = .1 => t = 1.363
Using [The P-values for the Popular Distributions](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/pvalues.htm%22%20%5Ct%20%22new) JavaScript, the 2p-value is:

P[| T |  1.363] = 0.2.

Question for you: Find t such that P( T8 > t ) = .01

**Applications:**

1. Testing hypotheses on mean, variance unknown
Given xbar16 = 12.1 and S2 = 2.225, test  = 12.5 vs  12.5, at  = 0.05 significance level.
The computed statistic is t = -1.07 but the critical value from t-table = 2.131.
Conclusion: There is no reason to reject that  = 12.5.

Question for you: Given the same sampling information perform the test H0:  = 11 vs  11 at  = .01.

2. Construction of confidence interval for , variance unknown
**Example:** Given xbar16 = 12.1, S2 = 2.225 develop a 95% confidence interval for 

P[xbar t( / 2, n 1)   S / n1/2  xbar + t ( / 2, n 1)   S / n1/2]  1 - 

Therefore P[11.31 ]  0.95 Again, notice the **Duality** between the test of hypothesis and confidence interval.

Question for you: Construct a 90% confidence interval for the same problem, is it wider than the other one, why or why not?

Notice that the T-density converges to the standard normal N(0, 1) as sample size gets larger. In fact the elements in the last row of t-table are the N(0,1) probabilities.

**[Chi-square Density Function](http://home.ubalt.edu/ntsbarsh/Business-stat/opre504.htm%22%20%5Cl%20%22rtChiSquare%22%20%5Ct%20%22new)**

A Fact: If X N(?, ), then the random variable

(n-1)S 2 / 2  2(n  1)

the parameter (n-1) =  is called degrees-of-freedom (d.f).
**Example:** if d.f. =  = 15, and  = 0.975 find the  2 value. From the  2 table, we get  2 =6.26
Using [The P-values for the Popular Distributions](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/pvalues.htm%22%20%5Ct%20%22new) JavaScript, the p-value is:

P[ 2  6.26] = 0.975.

**Applications:**

1. Tests of hypotheses on the variance of a normal population.

Given n = 16 and S2 = 2.22 test that 2 = 2.0 at  = .05. The sampling statistic is 20 = 16.65, however from the table, the critical values are 2( 15, .025) = 27.4884 and 2(15, .975) = 6.26
Conclusion: There is no reason to reject that 2 = 2.0

2. Interval estimation of the variance of a normal population

P[(n-1)S2 / 2 (,  /2)   (n-1)S2 / 2(,  /2)] 

**Example:** Given the same sampling information as above construct a 95% confidence interval for 
Plugging in the given information, you should get:
P[ 1.332 

Again, notice the **Duality** between the test of hypothesis and confidence interval.

Question for you: Given the same sampling information should we reject that  = 2.0? at  = .1
Note that 2(15, .05) = 8.55, and 2(15, .95) = 7.26.

**[F-Density Function](http://home.ubalt.edu/ntsbarsh/Business-stat/opre504.htm%22%20%5Cl%20%22rFisherF%22%20%5Ct%20%22new)**

A Fact: Consider two independent samples, one form two normal populations with known variance 21, and 22, then

(S12 / 12) / (S22 / 22) F(n 1  1, n2  1)

**Example:** Find F such that P[F8, 7  F] = .05 => The F value is F = 3.73

Notice: By now, you should have noticed that while every Statistical Table collected at the end of your textbook, provides the critical values for the right-tail as well as the left-tail probabilities, except the F-Table, which contains the critical values for the right-tail probabilities only. However, one might use the following nice property of F-distribution that:

F1, 2, 1-  = 1 / F2, 1, 

to obtain the critical values for the left-tail probabilities. Here is a numerical example:

F 2, 3, 0.9 = 1 / F 3, 2, 0.1 = 1 / 9.16 = 0.109

You need both tails probabilities for test of hypothesis and construction of confidence interval for the ratio of two independent populations' variances.

Example: Find P[F8, 7  F] = .95. We may not be able to get the critical value from the table, however, one may utilize the fact that:

F1, 2,  = 1 / F2, 1, 

Therefore, F = 1 / 3.50 = 0.2857
Using [The P-values for the Popular Distributions](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/pvalues.htm%22%20%5Ct%20%22new) JavaScript, the p-value is: P[ F  0.2857] = 0.942 (which is exact).

**Applications:**

1. Testing of hypothesis on the variance of two normal populations.

**Example:** Given n1 = n2 = 16, S12 = 34.14, and S22 = 47.32, should we reject that 12 = 22 at  = 0.1
The sampling statistics is F = S12 / S22 = .721, but the critical values are F15, 15, .05 = 2.38, and F15, 15, .95 = 1 / 2.38 =0.421.
Conclusion: Therefore is no reason to reject.

Question for you: Given the same sampling information, construct a 90% confidence interval for variance ratio: 12 / 22