# The Carpenter's Problem: Allocating Scarce Resources Among Competitive Means

During a couple of brain-storming sessions with a carpenter (our client), he told us that he, solely, makes tables and chairs, sells all tables and chairs he makes at a market place, however, does not have a stable income, and wishes to do his best.

The objective is to find out how many tables and chairs he should make to maximize net income. We begin by focusing on a time frame, i.e., planning time-horizon, to revise our solution weekly if needed. To learn more about his problem, we must go to his shop and observe what is going on and measure what we need to formulate (i.e., to give a Form, to make a model) of his problem. We must confirm that his objective is to maximize net income. We must communicate with the client.

The carpenter's problem deals with finding out how many tables and chairs to make per week; but first an objective function must be established:

Since the total cost is the sum of the fix cost (F) and the variable cost per unit multiplied by the number of units produced. Therefore, the decision problem is to find X1 and X2 such that:

 Maximize 9X1 + 6X2 – [(1.5X1 + X2) + (2.5X1 + 2X2) + F1 + F2],

where X1 and X2 stand for the number of tables and chairs; the cost terms in the brackets are the raw material, and labor costs respectively. F1 and F2 are the fixed costs for the two products respectively. Without loss of generality, and any impact on optimal solution, let us set F1 = 0, and F2 = 0. The objective function reduces to the following net profit function:

 Maximize 5X1 + 3X2

That is, the net incomes (say, in dollars, or tens of dollars) from selling X1 tables and X2 chairs.

The constraining factors which, usually come from outside, are the limitations on labors (this limitation comes from his family) and raw material resources (this limitation comes from scheduled delivery). Production times required for a table and a chair are measured at different times of day, and estimated to be 2 hours and 1 hour, respectively. Total labor hours per week are only 40 hrs. Raw materials required for a table and a chair are 1, and 2 units respectively. Total supply of raw material is 50 units per week. Therefore, the LP formulation is:

 Maximize 5 X1 + 3 X2,

 Subject to:

 2 X1 + X2 ≤ 40 labor constraint

 X1 + 2 X2 ≤ 50 material constraint

 and both X1, X2 are non-negative.