

DECISION MODELS

Decisions Making Under Uncertainty

1. Max-Min Criteria (pessimist)
 - Best of the worst
 - Becomes (Min-Max) if loss table
 - A. Find min (max) in each row
 - B. Pick the best of the Max (Min)

Criteria Max-Max Criterion (Optimist)

- Best of the best
- (min-min) if loss table
- A. Find max (min) in each row
- B. Pick the largest (smallest)

3. Weighted Average Criterion

- nature - Coef. of Optimism = α
- Optimistic=1, pessimistic=0
- A. Calculate weighted value expected
$$\alpha (\text{best}) + (1 - \alpha) (\text{worst})$$
 - B. Pick best value

4. Minimizing Regret

- Savage opportunity loss criteria
- A. Set up opportunity loss matrix
- subtract the largest number in each column from all other numbers in that column
- B. Find max regret in each row
- C. Pick the action with min. regret.

6. Equal Likely Strategy (Laplace Criterion)

- Best on Average
- A. Expected payoffs for each row
- B. Pick the largest (max problem) (Smallest for min problems)

Decisions Making Under Risk

1. Expected Payoff (Average)

- A. Multiply payoffs by probabilities and add up. (For each action separately)
- B. Pick best action

2. Expected Opportunity Loss

- A. Set up loss matrix
- Subtract all numbers in each column from the largest number in that column
- B. Find average opportunity loss Becomes for each action.
- multiply the probability time loss and B. add up.
- C. Pick smallest number (want to min loss)

3. Most Probable State of Nature

- A. Determine the most probable state of (one with highest probability)
- B. Pick the action with the highest payoff.
- C. Good criteria for a non-repetitive Decision

5. Expected Value of Perfect Information

- A. Fix a state of nature.
- B. Pick largest value in each column
- C. Multiply prob. X largest values and add up = ERPI
- ERPI=Expected Return of Perfect Info)
- D. EVPI = ERPI - Average expected payoff
- E. EVPI is always equal to Expected opportunity loss.

Numerical Example

	<u>State of Nature</u>			
	(0.5)**	(0.3)**	(0.2)**	**Use Probabilities for Decision Under Risk Problems only.
Action	<u>Growth</u>	<u>No Change</u>	<u>Inflation</u>	
Bonds	12%	6%	3%	
Stocks	15%	3%	-2%	
Deposit	6.5%	6.5%	6.5%	

Note: Objective is to Maximize

DECISION MAKING UNDER PURE UNCERTAINTY

1. <u>Max-Min (Pess)</u>	2. <u>Max-Max (Opt)</u>	3. <u>Weighted Average</u>
<u>Min/Row</u>	<u>Max/Row</u>	<u>Action</u> <u>Weighted Value ($\alpha = 0.7$)</u>
3	12	B $(.7) 12 + (.3) 3 = 9.3$
-2	15 **	S $(.7) 15 + (.3) -2 = 9.9 **$
6.5 **	6.5	D $(.7) 6.5 + (.3) 6.5 = 6.5$
		Best Worst

4. Minimizing Regret

5. Equal Likely Strategy (LaPlace)

<u>Opportunity Loss Matrix</u>						<u>Action</u>
<u>Action</u>	<u>Growth</u>	<u>No Change</u>	<u>Inflation</u>	<u>Max/Row</u>		<u>7 **</u>
Bonds	-3 (12-15)	-0.5	-3.5	-3.5 **	Bonds**	
Stocks	0	-3.5	-8.5	-8.5	Stocks	5.3
Dep.	-8.5	0	0	-8.5	Deposit	6.5

DECISION MAKING UNDER RISK

1. Expected Payoff (Average)

<u>Action</u>	<u>Average Payoff</u>
** Bonds	$(.5)12 + (.3)6 + (.2)3 = 8.4 **$
Stocks	$(.5)15 + (.3)3 + (.2)-2 = 8.0$
Deposit	$(.5)6.5 + (.3)6.5 + (.2)(6.5) = 6.5$

2. Expected Opportunity Loss

<u>Opportunity Loss (EOL) Matrix</u>				
<u>Act</u>	<u>G (.5)</u>	<u>No (.3)</u>	<u>In (.2)</u>	<u>EOL</u>
B**	3 (15-12)	0.5	3.5	2.35**
S	0	3.5	8.5	2.75
D	8.5	0	0	4.25

3. Most Probable State of Nature

<u>Action</u>	<u>Growth (.5)</u>	
Bonds	12	Note: EOL is the sum of the (prob.* loss)
Stocks	15**	$3(.5) + .5(.3) + 3.5(.2) = 2.35$
Deposit	6.5	$0(.5) + 3.5(.3) + 9.5(.2) = 2.75$
		$8.5(.5) + 0(.3) + 0(.2) = 4.25$

4. Expected Value of Perfect Information

EVPI = ERPI - Average Expected Payoff

Max Values from Each Column			
<u>Growth(.5)</u>		<u>No Change(.3)</u>	<u>Inflation(.2)</u>
15		6.5	6.5
ERPI= 15(.5)	+	6.5(.3)	+ 6.5(.2) = 10.75

$$EVPI = 10.75 - 8.4 = 2.35\%$$

If information costs more than 2.35%, don't buy it. If you invest \$100,000 should you buy info for \$15,000? $2.35\% (\$100,000) - \$15,000 = -\$12,650 \Rightarrow \text{NO!}$

DECISION TREES (Bayesian Approach)

1. Evaluate the Decision with Prior Probabilities

Action	<u>State of Nature</u>		
	<u>A (High Sales) (.2)</u>	<u>B (Medium Sales) (.5)</u>	<u>C (No Sales) (.3)</u>
A1 (Develop)	3000	2000	-6000
A2 (Don't)	0	0	0

Prior EMV : Develop: $(.2)3000 + (.5)2000 + (.3)(-6000) = -200$

Prior EMV (Don't): 0 **

2. Acquire Some Reliable Info (Not Perfect Info Due To Uncertainty)

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Predicted	<u>A (High)</u>	<u>B (Medium)</u>	<u>C (Small)</u>	Consultant is best at Predicting medium sales.
Ap	0.8	0.1	0.1	
Bp	0.1	0.9	0.2	
Cp	<u>0.1</u>	<u>0.0</u>	<u>0.7</u>	
Sum	1.0	1.0	1.0	

3. Revised (Posterior) Probabilities are Computed

<u>State of Nature</u>	<u>Predictions</u>				<u>Prior Prob.</u>	<u>Ap . P Bp . P Cp . P </u>		
	<u>Ap</u>	<u>Bp</u>	<u>Cp</u>					
A	.8	.1	.1		.2	.16 .02 .02		
B	.1	.9	0		.5	.05 .45 0		
C	.1	.2	.7		.3	.03 .06 .21		
Sum						.24 .53 .23	add-up to 1	
0.2 = P(Bp C)						.16/.24 .02/.53 .02/.23		
Note: Table is inverted, now						= .667 = .038 = .087		
rows add to equal 1.						.05/.24 .45/.53 0/.23		
See decision tree for use of values						= .208 = .849 = 0		
0.113 = P(C Bp)						.03/.24 .06/.53 .21/.23		
						= .125 = .113 = .913		
Sum						1 1 1		

4. Expected Values Are Computed: See decision tree

5. A decision is made regarding whether or not to acquire the additional info. Then a choice is made immediately.

6. If a decision is made to buy the info, then the research is undertaken only after that, based on the results of the research, is the selection of an alternative made.

7. Expected Value of Perfect Information

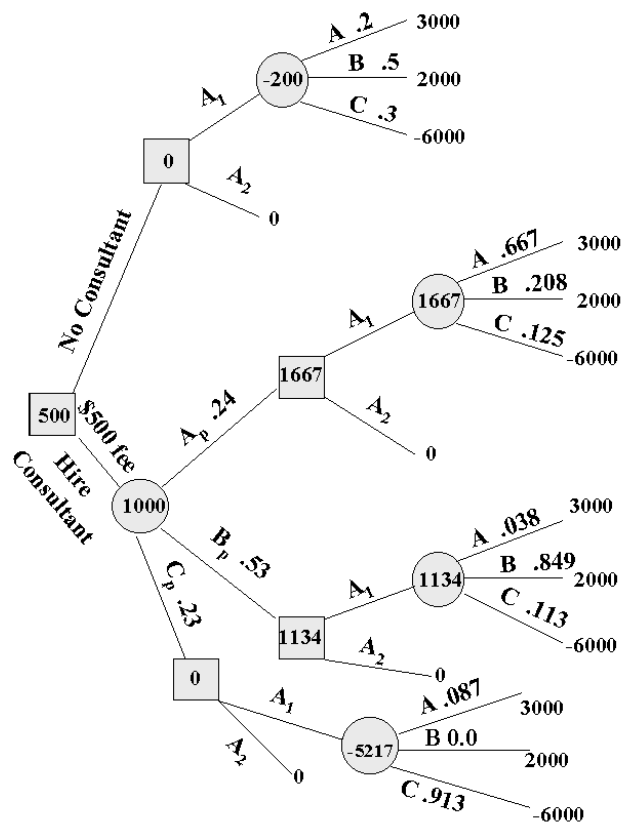
EVPI = EMV (with devil help) - EMV (without devil help)

EMV (without his help) = 0

EVPI = $(.2)3000 + (.5)2000 + (.3)0 - 0 = 1600$

Best outcomes for each state of nature.

Efficiency of the Consultant = Expected Payoff (using consultant)/EVPI = $1000/1600 = 62.5\%$



UTILITIES: Utility: Value of \$ to you based on your risk profile.

	<u>Fire (.0005)</u>	<u>No Fire (.9995)</u>	<u>Expected Utility</u>
Insure	-1	-1	-1 **
Don't Ins.	-10,000 0	-5	

How to find Utility of \$12

= $(P) \times \text{Utility of } \$15 + (1-P) \times \text{Utility of } -\2

- Find "P" where you are indifferent. Once you have few points, graph and interpolate all other utilities.