

Wilson Problem:

Max $7X_1 + 10X_2$

Subject to:

$5X_1 + 6X_2 \leq 3600$ square feet of cowhide

$1X_1 + 2X_2 \leq 960$ minutes of production time

$X_1 \leq 500$ baseballs produced daily

$X_2 \leq 500$ softballs produced daily

$X_1, X_2 \geq 0$ non-negativity conditions (implied constraints)

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 5520.000

VARIABLE	VALUE	REDUCED COST
X1	360.000000	0.000000
X2	300.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	2.000000
4)	140.000000	0.000000
5)	200.000000	0.000000

NO. ITERATIONS= 2

The optimal solution for the primal problem is $X_1 = 360$, $X_2 = 300$, optimal values of \$5520. Dual prices column indicating that they are the optimal solution for the Dual problem, which means they are, shadow Prices (of each RHS). $U_1 = 1$, $U_2 = 2$, $U_3 = 0$, $U_4 = 0$.

RANGES IN WHICH THE BASIS IS UNCHANGED: The following part provides the current coefficients values of the objective function and the range for each that the change in each coefficient for which the optimal solution remain the same.

VARIABLE	OBJ COEFFICIENT RANGES		
	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	7.000000	1.333333	2.000000
X2	10.000000	4.000000	1.600000

RIGHTHAND SIDE RANGES: This part provides the current RHS of constraints values and the range for each that the change in each RHS for which the solution to the dual problem (the shadow prices) remain the same.

ROW	CURRENT	ALLOWABLE	ALLOWABLE
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	RHS	INCREASE	DECREASE
2	3600.000000	280.000000	720.000000
3	960.000000	160.000000	93.333336
4	500.000000	INFINITY	140.000000
5	500.000000	INFINITY	200.000000

Some “What IF” Analysis

A) Change of Objective Function Coefficient:

According to Lindo’s output, the optimal solution does not change , that is X1’s coefficient of 7 can increase by 1.3 and decrease by 2. X2’s coefficient of 10 can increase by 4 and decrease by 1.6. So, the optimal solution remains the same whenever X1’s coefficient can have a value range of 5-8.3, while X2’s coefficient can have a value range of 8.4-14. However any change within these ranges changes the optimal values. The change can be computed by plugging in the current optimal solution into the new objective function with new coefficient (one-change-at-a-time, only)

1) Changing X1 with Lindo output:

X1 coefficient = 6
(6X1 + 10X2)

LP OPTIMUM FOUND AT STEP 0

OBJECTIVE FUNCTION VALUE

1) 5160.000

VARIABLE	VALUE	REDUCED COST
X1	360.000000	0.000000
X2	300.000000	0.000000

Still remains optimal.

2) X2 coefficient= 13
(7X1 + 13X2)

LP OPTIMUM FOUND AT STEP 0

OBJECTIVE FUNCTION VALUE

1) 6420.000

VARIABLE	VALUE	REDUCED COST
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X1	360.000000	0.000000
X2	300.000000	0.000000

Still remains unchanged.

Optimal values will change, as expected.

B) Change of RHS values (one-change-at-a-time)

Suppose we change 3600 to 3700 that is with the range of RHS of first constraint. The optimal value changes proportion to its shadow price. That is \$1 (3700-3600) = \$100 increase 5520.000 + 100 = 5620. Which can be verified by running LINDO for the changed problem.

More What if Analysis:

1) Deletion of Constraint- $1X1 + 2X2 \leq 960$ (minutes of production time)

Since this constraint is a binding constraint (having slack = 0), it is an important constraint passing through the optimal vertex because $1(360) + 2(300) = 960$. Therefore the optimal solution may change.

To find the change we have to run LINDO for this new problem:

Max $7X1 + 10X2$

Subject to:

$5X1 + 6X2 \leq 3600$ square feet of cowhide

$X1 \leq 500$ baseballs produced daily

$X2 \leq 500$ softballs produced daily

$X1, X2 \geq 0$ non-negativity conditions (implied constraints)

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 5840.000

VARIABLE	VALUE	REDUCED COST
X1	120.000000	0.000000
X2	500.000000	0.000000

As expected the optimal solution is changed, therefore the optimal value also changed. Since this problem is a two-dimensional LP, it can also be verified by Graphical Method:

Enter the linear programming problem here:

☒ Maximize $z = 7x + 10y$ subject to the constraints:
☐ Minimize
☐ Show only the region defined by the following constraints:

$5x + 6y \leq 3600$
 $x \leq 500$
 $y \leq 500$

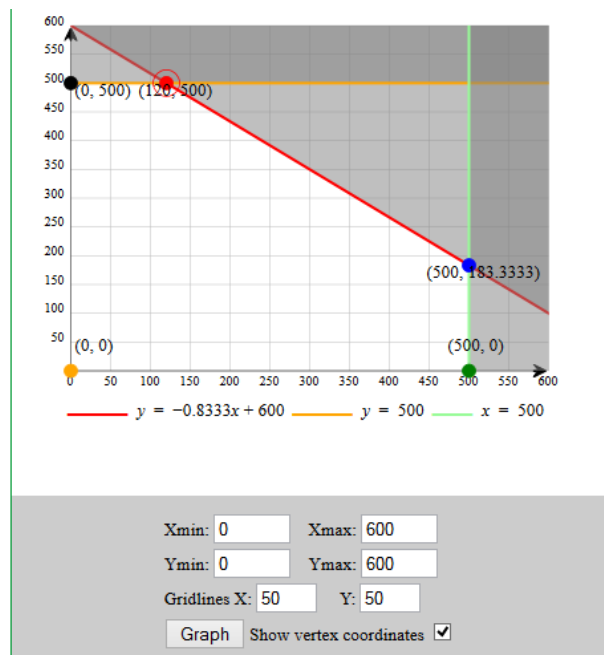
LP Examples Graphing Examples **Solve**

Rounding: 4 decimal places Fraction Mode ☐

Erase Everything

The solution will appear below.

Vertex	Lines through vertex	Value of objective
● (500, 183.3333)	$5x + 6y = 3600$ $x = 500$	5333.3333
● (120, 500)	$5x + 6y = 3600$ $y = 500$	5840 Maximum
● (500, 0)	$x = 500$ $y = 0$	3500
● (0, 500)	$y = 500$ $x = 0$	5000



2) Addition of a Variable- X3- addition of baseball gloves at 4 minutes of manufacturing time, 2sq ft of cowhide, and \$19 in profit.

max $7x_1 + 10x_2 + 19x_3$
 st
 $5x_1 + 6x_2 + 2x_3 \leq 3600$
 $x_1 + 2x_2 + 4x_3 \leq 960$
 $x_1 \leq 500$
 $x_2 \leq 500$
 $x_3 \leq 500$

The question is whether the new product is profitable? To find out one must use the shadow prices. Ask how much it cost to produce one unit of new product? $\$1 (2) + \$2 (4) = \$10$, however the profit is \$19 therefore it is profitable. The question now is how much to produce. For this we have to use Lindo for this three-dimensional problem (There is no way to use the graphical method):

LP OPTIMUM FOUND AT STEP 3

OBJECTIVE FUNCTION VALUE

1) 5772.000

VARIABLE	VALUE	REDUCED COST
X1	500.000000	0.000000
X2	174.000000	5520.000
X3	28.000000	0.000000

**Optimal Value: $z = 5772$; and
the Optimal Solution (optimal Strategy) $x_1 = 500$, $x_2 = 174$, $x_3 = 28$**

3) Addition of Constraint- $x_1 + x_2 \leq 600$: Wilson is limited to production of 600 units of both baseballs and softballs

The current solution $x_1 = 360$, $x_2 = 300$, does not satisfy this constraint. Therefore, we have to redo Lindo with this add constraint:

Max $7x_1 + 10x_2$

Subject to:

$5x_1 + 6x_2 \leq 3600$ square feet of cowhide

$1x_1 + 2x_2 \leq 960$ minutes of production time

$x_1 + x_2 \leq 600$ baseballs and softballs produced daily

$x_1 \leq 500$ baseballs produced daily

$x_2 \leq 500$ softballs produced daily

$x_1, x_2 \geq 0$ non-negativity conditions (implied constraints)

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) **5280.000**

VARIABLE	VALUE	REDUCED COST
X1	240.000000	0.000000
X2	360.000000	0.000000

4) Deletion of a Variable x_2

One must redo to find the new optimal value (which will be smaller, because Wilson lost the opportunity of making more \$) the optimal solution, clearly changes, because x_2 is set to zero.

Max $7x_1$

Subject to:

$5x_1 \leq 3600$ square feet of cowhide

$x_1 \leq 960$ minutes of production time

$x_1 \leq 500$ baseballs produced daily

$x_1 \geq 0$ non-negativity conditions (implied constraints)

LP OPTIMUM FOUND AT STEP 1

OBJECTIVE FUNCTION VALUE

1) 3500.000

VARIABLE	VALUE	REDUCED COST
X1	500.000000	0.000000

Graph:

