The Carpenters Problem and Its Dual

Maximize $5 \times X_1 + 3 \times X_2$
Subject to:
$2 \times X_1 + X_2 \leq 40$ labor constraint
$X_1 + 2 \times X_2 \leq 50$ material constraint
and both $X_1, X_2$ are non-negative.

Introducing Slacks/Surplus variables we have,

Maximize $5 \times X_1 + 3 \times X_2$
Subject to:
$2 \times X_1 + X_2 + S_1 = 40$
$X_1 + 2 \times X_2 + S_2 = 50$
and both $X_1, X_2, S_1, S_2$ are non-negative.

The Final Simplex Tableau for the Primal is:

<table>
<thead>
<tr>
<th>BVS</th>
<th>X1</th>
<th>X2</th>
<th>S1</th>
<th>S2</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>1</td>
<td>0</td>
<td>2/3</td>
<td>-1/3</td>
<td>10</td>
</tr>
<tr>
<td>X2</td>
<td>0</td>
<td>1</td>
<td>-1/3</td>
<td>2/3</td>
<td>20</td>
</tr>
<tr>
<td>Cj</td>
<td>0</td>
<td>0</td>
<td>-7/3</td>
<td>-1/3</td>
<td>110</td>
</tr>
</tbody>
</table>

The Dual Problem for the Carpenter:

Minimize $40 \times U_1 + 50 \times U_2$
Subject to:
$2U_1 + U_2 \geq 5$ Net Income from a table
$U_1 + 2U_2 \geq 3$ Net Income from a chair
and $U_1, U_2$ are non-negative.

Introducing Slacks/Surplus variables we have,

Minimize $40 \times U_1 + 50 \times U_2$
Subject to:
$2U_1 + U_2 - S_1 = 5$
$U_1 + 2U_2 - S_2 = 3$
and $U_1, U_2, S_1, S_2$ are non-negative.

The Final Simplex Tableau for the Dual is:

<table>
<thead>
<tr>
<th>BVS</th>
<th>U1</th>
<th>U2</th>
<th>S1</th>
<th>S2</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>U2</td>
<td>0</td>
<td>1</td>
<td>1/3</td>
<td>-2/3</td>
<td>1/3</td>
</tr>
<tr>
<td>U1</td>
<td>1</td>
<td>0</td>
<td>-2/3</td>
<td>1/3</td>
<td>7/3</td>
</tr>
<tr>
<td>Cj</td>
<td>0</td>
<td>0</td>
<td>-10</td>
<td>-20</td>
<td>110</td>
</tr>
</tbody>
</table>