1. National Foods' ad agency has constructed the following payoff table giving its estimate of the expected profit resulting from purchasing one, two, or three advertising sports. (Another possible decision is for National Foods not to advertise at all during the Super Bowl.) The states of nature correspond to the game being "dull", "average", above average", or "exciting".
a. What is the optimal decision if the National Foods advertising manager is optimistic?

| Number of 30-second <br> Commercials <br> Purchased |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Perceived Game Excitement |  |  |  |  |
|  | Dull | Average | Above <br> average | Exciting | Optimism |
| One | -2 | 3 | 7 | 13 | 13 |
| Two | -5 | 6 | 12 | 18 | 18 |
| Three | -9 | 5 | 13 | 22 | $22 * *$ |

The optimal decision (the maximum) if the national foods advertising manager is optimistic is 22 . In the best case scenario, managers will purchase 3 thirtysecond commercials.
b. What is the optimal decision if the National Foods advertising manager is pessimistic?

| Number of 30-second <br> Commercials <br> Purchased |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Perceived Game Excitement |  |  |  |  |
|  | Dull | Average | Above <br> average | Exciting | Pessimistic |
| One | -2 | 3 | 7 | 13 | $-2 * *$ |
| Two | -5 | 6 | 12 | 18 | -5 |
| Three | -9 | 5 | 13 | 22 | -9 |

The optimal decision (the minimum) if the National Food advertising manager is pessimistic is -2 . In the worst case scenario, Managers will purchase 1 thirtysecond commercial. They can also purchase 0 commercials, but that will lead to 0 profits.
c. What is the optimal decision if the National Foods advertising manager wishes to minimize the firm's maximum regret?

| Number of 30-second <br> Commercials <br> Purchased |  |  |  |  |  |
| :--- | :---: | :---: | :--- | :--- | :---: |
|  | Perceived Game Excitement |  |  |  |  |
|  | Dull | Average | Above <br> average | Exciting | Maximum <br> regret |
| One | $-2+2=\mathbf{0}$ | $6-3=\mathbf{3}$ | $13-7=\mathbf{6}$ | $22-13=\mathbf{9}$ | 9 |
| Two | $-2+5=\mathbf{3}$ | $6-6=\mathbf{0}$ | $13-12=\mathbf{1}$ | $22-18=\mathbf{4}$ | $4 * *$ |
| Three | $-2+9=\mathbf{7}$ | $6-5=\mathbf{1}$ | $13-13=\mathbf{0}$ | $22-22=\mathbf{0}$ | 7 |

If the manager wishes to minimize the firm's maximum regret, then they much purchase 2 thirty-second commercials; the minimum of these maximum regret is 4 .
2. Consider the data given in problem 1 for National Foods. Based on past Super Bowl games, suppose the decision maker believes that the following probabilities hold for the states of nature:
$\mathrm{P}($ Dull game $)=.20$
$\mathrm{P}($ Average Game $)=.40$
$\mathrm{P}($ Above Average Game $)=.30$
$\mathrm{P}($ Exciting Game $)=.10$
a. Using the expected value criterion, determine how many commercials National Foods should purchase.

| Number of 30 -second Commercials Purchased | Perceived Game Excitement |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Dull $(0.20)$ | $\begin{aligned} & \text { Average } \\ & (0.40) \end{aligned}$ | Above average (0.30) | Exciting <br> $(0.10)$ | Expected value |
| One | $\begin{gathered} -2(.20)= \\ -.4 \end{gathered}$ | $\begin{gathered} 3(.40)= \\ 1.2 \end{gathered}$ | $7(.30)=2.1$ | $\begin{gathered} 13(.10)= \\ 1.3 \end{gathered}$ | 4.2 |
| Two | $-5(.20)=$ | $\begin{gathered} 6(.40)= \\ 2.4 \end{gathered}$ | $\begin{gathered} 12(.30)= \\ 3.6 \end{gathered}$ | $\begin{gathered} 18(.10)= \\ 1.8 \end{gathered}$ | 6.8** |
| Three | $\begin{gathered} -9(.20)= \\ -1.8 \end{gathered}$ | $5(.40)=2$ | $\begin{gathered} 13(.30)= \\ 3.9 \end{gathered}$ | $\begin{gathered} 22(.10)= \\ 2.2 \end{gathered}$ | 6.3 |

Commercials nation foods should purchase two 30 -second commercials. This decision will be optimal over the long run.
b. Based on the probabilities given here, determine the expected value of perfect information.

The expected return with perfect information is $.20(-2)+.40(6)+.30(13)+.10(22)=8.1$. So the expected return with perfect information is $\mathbf{E V I P}=8.1 \mathbf{- 6 . 8}=\mathbf{1 . 3}$. This represents the gain in expected return resulting by knowing the probabilities of the ratings of the Super Bowl games. Therefore, if the information costs more than $1.3 \%$ of the commercial, don't buy it.

3. Consider the data given in problem 2 and 3 for the National Foods. The firm can hire the noted sport's pundit Jim Worden to give his opinion as to whether or not the Super Bowl game will be interesting. Suppose the following probabilities hold for Jim's predictions:
$\mathrm{P}(\mathrm{Jim}$ predicts game will be interesting | game is dull $)=.15$
$P$ (Jim predicts game will be interesting | game is average $=.25$
$P$ (Jim predicts game will be interesting | game is above average $=.50$
$P$ (Jim predicts game will be interesting | game is exciting $=.80$
$P(\operatorname{Jim}$ predicts game will not be interesting | game is actually dull $)=.85$
$P($ Jim predicts game will not be interesting | game is average $)=.75$
$P(\operatorname{Jim}$ predicts game will not be interesting | game is above average) $=.50$
$P($ Jim predicts game will not be interesting | game is exciting $)=.20$
a. Jim predict the game will be interesting, what is the If probability the game will be dull?

IF the game is interesting

| Game <br> excitement | Prior <br> probabilities | Conditional <br> probabilities | Combined <br> probabilities | Posterior <br> probabilities |
| :--- | :--- | :--- | :--- | :--- |
| Dull | .20 | .15 | $.20(.15)=$ <br> .03 | $.03 / .36=.08^{* *}$ |
| Average | .40 | .25 | $.40(.25)=$ <br> .10 | $.10 / .36=.28$ |
| Above <br> average | .30 | .50 | $.30(.50)=$ <br> .15 | $.15 / .36=.42$ |
| Exciting | .10 | .80 | $.10(.80)=$ | $.08 / .36=.22$ |
|  |  |  | .08 |  |

There is an $8 \%$ probability that the game will be dull if Jim predicts that the game will be interesting.

IF the game is not interesting

| Game excitement | Prior <br> probabilities | Conditional <br> probabilities | Combined <br> probabilities | Posterior <br> probabilities |
| :--- | :--- | :--- | :--- | :--- |
| Dull | .20 | .85 | $.20(.85)=.17$ | $.17 / .64=.27$ |
| Average | .40 | .75 | $.40(.75)=.3$ | $.3 / .64=.47$ |
| Above average | .30 | .50 | $.30(.50)=.15$ | $.15 / .64=.23$ |
| exciting | .10 | .20 | $.10(.20)=.02$ | $.02 / .64=.04$ |
| Total |  |  |  | 0.64 |

b. What is the national's strategy if Jim predicts the game will be (i) Interesting or (ii) not interesting?

| Number of $30-$ second Commercials Purchased | Perceived Game Excitement |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{array}{\|l\|} \hline \text { Dull } \\ (.08) \end{array}$ | $\begin{aligned} & \text { Average } \\ & (.28) \end{aligned}$ | Above average (.42) | $\begin{aligned} & \text { Exciting } \\ & \text { (.22) } \end{aligned}$ | $\begin{aligned} & \mathrm{EV} \\ & \text { (interesting) } \end{aligned}$ |
| One | $\begin{gathered} -2(.08)= \\ -.16 \end{gathered}$ | $\begin{gathered} 3(.28)= \\ .84 \\ \hline \end{gathered}$ | $\begin{gathered} 7(.42)= \\ 2.94 \end{gathered}$ | $\begin{gathered} 13(.22)= \\ 2.86 \end{gathered}$ | 6.48 |
| Two | $\begin{gathered} -5(.08)= \\ -.4 \end{gathered}$ | $\begin{gathered} 6(.28)= \\ 1.68 \end{gathered}$ | $\begin{gathered} 12(.42)= \\ 5.04 \end{gathered}$ | $\begin{gathered} 18(.22)= \\ 3.96 \end{gathered}$ | 10.28 |
| Three | $\begin{gathered} -9(.08)= \\ -.72 \end{gathered}$ | $\begin{gathered} 5(.28)= \\ 1.4 \end{gathered}$ | $\begin{gathered} 13(.42)= \\ 5.46 \end{gathered}$ | $\begin{gathered} 22(.22)= \\ 4.84 \end{gathered}$ | $10.98{ }^{* *}$ |

i. If Jim predicts the game will be interesting, the optimal solution will be to purchase three 30 -second commercials.

| Number of 30second Commercials Purchased | Perceived Game Excitement |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{array}{\|l} \hline \text { Dull } \\ (.27) \end{array}$ | Average (.47) | $\begin{array}{\|l} \hline \begin{array}{l} \text { Above } \\ \text { average } \\ (.23) \end{array} \end{array}$ | $\begin{array}{\|l} \left\lvert\, \begin{array}{l} \text { Exciting } \\ (.04) \end{array}\right. \\ \hline \end{array}$ | EV (not interesting) |
| One | $\begin{gathered} -2(.27)= \\ -.54 \\ \hline \end{gathered}$ | $\begin{gathered} 3(.47)= \\ 1.41 \\ \hline \end{gathered}$ | $\begin{gathered} 7(.23)= \\ 1.61 \\ \hline \end{gathered}$ | 13(.04)= . 52 | 3 |
| Two | $\begin{gathered} -5(.27)= \\ -1.35 \end{gathered}$ | $\begin{gathered} 6(.47)= \\ 2.82 \end{gathered}$ | $\begin{gathered} 12(.23)= \\ 2.76 \end{gathered}$ | 18(.04)= . 72 | 4.95** |
| Three | $\begin{gathered} -9(.27)= \\ -2.43 \end{gathered}$ | $\begin{gathered} 5(.47)= \\ 2.35 \end{gathered}$ | $\begin{gathered} 13(.23)= \\ 2.99 \end{gathered}$ | 22(.04)= .88 | 3.79 |

ii. If Jim predicts the game will not be interesting, than the national optimal solution would be to purchase two 30 -second commercials.
c. What is the expected value of Jim's information?

The expected return with the sample information is $10.98(.36)+4.95(.64)=7.12$. So 7.12 -6.8 (expected value in problem 2) is equal to the expected value of Jim's information is about $\mathbf{0 . 3 2}$.
4. Steve Greene is considering purchasing fire insurance for his home. According to statistics for Steve's county, Steve estimates the damage from fire to his home in a given year is as follows:

| Amount of Damage | Probability |
| :---: | :--- |
| 0 | .975 |
| $\$ 10,000$ | .010 |
| $\$ 20,000$ | .008 |
| $\$ 30,000$ | .004 |
| $\$ 50,000$ | .002 |
| $\$ 100,000$ | .001 |

a. If Steve is risk neutral, how much should he be willing to pay for the fire insurance?

| Amount of Damage |  | Probability |  |
| :---: | :---: | :---: | :--- |
| 0 | $X$ | .975 | $=0$ |
| $\$ 10,000$ | $X$ | .010 | $=100$ |
| $\$ 20,000$ | $X$ | .008 | $=160$ |
| $\$ 30,000$ | $X$ | .004 | $=120$ |
| $\$ 50,000$ | $X$ | .002 | $=100$ |
| $\$ 100,000$ | $X$ | .001 | $=\underline{100}$ |
|  |  |  | Total: $\mathbf{5 8 0}$ |

Steve should be willing to pay \$580 for the fire insurance
Suppose Steve's utility values are as follows:

|  | Amount of Loss (\$1000s) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 100 | 50 | 30 | 20 | 10 | 1 | 0 |
| Utility | 0 | .65 | .75 | .8 | .95 | .995 | 1 |

b. What is the expected utility corresponding to fire damage?
$0.975(1)+0.010(.95)+0.008(.8)+0.004(.75)+0.002(.65)+0.001(0)=.9952$
The expected utility corresponding to fire damage is 0.9952 .
c. Determine approximately how much Steve would be willing to pay for the fire insurance?

Since Steve's expected utility is 0.9952 which is equals to 1 in the chart above, Steve should be willing to pay about $\$ 1000$ for the fire insurance. This amount is almost doubled from this risk neutral amount of $\$ 580$.

