Sensitivity Analysis

Sensitivity Analysis is used to see how the optimal solution is affected by the objective function coefficients and to see how the optimal value is affected by the right-hand side values. Using LINDO, you can figure out how to integrate different variables to the problem without affecting the objective function. The Wilson Problem's objective function was to maximize 7X1+10X2 and was subject to X1 < 500, X2 < 500, X1 + 2X2 < 960, and 5X1 + 6X2 < 3600. The optimal value for the Wilson problem was $5,520.

LINDO INPUT & RESULTS

Maximize 7X1 + 10X2

Subject to

X1 < 500

X2 < 500

X1 + 2X2 < 960

5X1 + 6X2 < 3600

END

(LINDO assumes < is less than or equal to)

(Click Solve on the top menu or ctrl +S)

(Asks: DO RANGE SENITIVITY ANALYSIS? YES)

(LINDO Solver Status)

(Optimizer Status)

(Status: Optimal)

(Iterations: 2)

(Objective: 5520)

NO. ITERATIONS= 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	7.000000	1.333333	2.000000
X2	10.000000	4.000000	1.600000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	500.000000	INFINITY	140.000000
3	500.000000	INFINITY	200.000000
4	3600.000000	280.000000	720.000000
5	960.000000	160.000000	93.333336

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 5520.000

VARIABLE	VALUE	REDUCED COST
X1	360.000000	0.000000
X2	300.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	140.000000	0.000000
3)	200.000000	0.000000
4)	0.000000	1.000000
5)	0.000000	2.000000

NO. ITERATIONS= 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	7.000000	1.333333	2.000000
X2	10.000000	4.000000	1.600000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	500.000000	INFINITY	140.000000
3	500.000000	INFINITY	200.000000
4	3600.000000	280.000000	720.000000

5 960.000000 160.000000 93.333336

Now to construct a dual problem, you would need to formulate a minimization linear model. For the Wilson problem we will use the decision variables U1, U2, U3, and U4, since there are 4 constraints in the original problem. The dual model for the Wilson problem would be:

Minimize 3600U1+960U2+500U3+500U4

Subject to

5U1+1U2+U3>=7

6U1+2U2+U4>=10

Uj>=0

END

As you can see we have switched the objective function to the constraints and the constraints to the object function. This would be the dual problem for the Wilson Problem. After we have figured out our program model, we can use LINDO in order to solve the problem.

LINDO INPUT

Minimize 3600U1+960U2+500U3+500U4

Subject to

5U1+1U2+U3>=7

6U1+2U2+U4>=10

Uj>=0

END

LINDO RESULTS

LP OPTIMUM FOUND AT STEP 3

OBJECTIVE FUNCTION VALUE

1) 5520.000

VARIABLE	VALUE	REDUCED COST
U1	1.000000	0.000000
U2	2.000000	0.000000
U3	0.000000	140.000000
U4	0.000000	200.000000
UJ	0.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-360.000000
3)	0.000000	-300.000000
4)	0.000000	0.000000

NO. ITERATIONS= 3

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
U1	3600.000000	280.000000	720.000000
U2	960.000000	160.000000	93.333336
U3	500.000000	INFINITY	140.000000

U4	**500.000000**	**INFINITY**	**200.000000**
UJ	**0.000000**	**INFINITY**	**0.000000**

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	**7.000000**	**1.333333**	**2.000000**
3	**10.000000**	**4.000000**	**1.600000**
4	**0.000000**	**0.000000**	**INFINITY**

As you can see the objective coefficients from the original problem have become the right hand side ranges in the dual problem. The results from the minimization problem have shown that the optimal value is the same, $5,520. The solutions being U1=1, U2=2, U3=0, U4=0, were found at Step 3. The value column comes from the Right Hand Side Ranges. The slack or surplus columns result from remaining or excess resources. The cost coefficient can increase or decrease without affecting the current optimal solution of 5520. That is what either output is called combined report: Final report to the primal and the dual problems.

Definitions of LINDO Terms Related to Original and Dual Wilson Problem

LP Optimum: Tells which step the optimal solution was found at. The original Wilson problem was found at step 2 and the dual problem was found at step 3.

Objective Function Value: The value to which the decision variables were used to optimize the objective function. The objective function value for the Wilson problem and dual problem was $5,520, it's the same as we expect.

Variable: In the original Wilson problem the decision variables were X1 and X2, but in the dual problem they were U1, U2, U3 and U4. These variables represent the objectives being produced.

Value: The values of the decision variables. In the original problem, X1=360 and X2=300, and in the dual problem U1=1, U2=2, U3=0, and U4=0. When plugged in to the objective function you will find the optimal value of the objective function.

Reduced Cost: These values indicate the amount the coefficient value in the objective function must increase before it has a positive impact on the optimal solution. In both problems, the values is 0, because solution to both problem are positive numbers.

Slack or Surplus: Represents the lack of or excess of resources from production.

Shadow Price: The amount of change to the right hand side of a constraint and is also the solution to the dual problem. For example, the dual prices in the original Wilson problem were 0, 0, 2, 1 and are the solution for the dual problem being U1=1, U2=2, U3=0, and U4=0. The dual prices in the dual problem were 360 and 300, which are the solutions for the optimal value for X1 and X2.

What if Analysis

1) For the first what if analysis, I decided to delete a constraint. The constraint I deleted was the amount of cowhide allowed for production of baseballs and softballs. This constraint is binding which means it will change the optimal solution. The deletion of this constraint yielded a surplus of 270 for softballs. It takes less time to produce baseballs, so with no limit to the amount of cowhide for production, Wilson would utilize production of baseballs. The optimal solution of the deleted constraint would maximize profit at $5800, with production of 500 baseballs, and 230 softballs. The LINDO results were:

    Maximize 7X1 + 10X2

    Subject to

    X1 < 500

    X2 < 500

    X1 + 2X2 < 960

    END

    LP OPTIMUM FOUND AT STEP 1

    OBJECTIVE FUNCTION VALUE

    1) 5800.000

VARIABLE	VALUE	REDUCED COST
X1	500.000000	0.000000
X2	230.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	2.000000
3)	270.000000	0.000000
4)	0.000000	5.000000

NO. ITERATIONS= 1

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	7.000000	INFINITY	2.000000
X2	10.000000	4.000000	10.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE

2	500.000000	460.000000	500.000000
3	500.000000	INFINITY	270.000000
4	960.000000	540.000000	460.000000

2) For the second what if analysis, I decided to add a variable. I decided to add the production of a cricket ball. Cricket balls are similar to baseballs with a core and the use of hide for the outer cover. Since there will be no change to the amount of cowhide or time available for production, there will probably be no change to the objective function. The profit per cricket ball will be $5 and the maximum amount of cricket balls produced daily will be 300. It will take 3 sq. ft. of hide per dozen cricket balls and take 90 seconds to produce each dozen. After solving the new variable in LINDO, the optimal solution did not change and showed that the production of the cricket ball is not necessary. That is, it is not profitable to produce the new product.

One can see that by considering what goes into the new product 1.5($1) + 3($2) = $7.5 which more that net profit of 5. Therefore do not product. The following report confirms it.

The objective coefficient ranges remained the same except for the allowable decrease of softballs. LINDO results:

Maximize 7X1 + 10X2 + 5X3

Subject to

X1 < 500

X2 < 500

X3 < 300

X1 + 2X2 + 1.5X3< 960

5X1 + 6X2 + 3X3 < 3600

END

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 5520.000

VARIABLE	VALUE	REDUCED COST
X1	360.000000	0.000000
X2	300.000000	0.000000
X3	0.000000	1.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	140.000000	0.000000
3)	200.000000	0.000000
4)	300.000000	0.000000
5)	0.000000	2.000000
6)	0.000000	1.000000

NO. ITERATIONS= 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	7.000000	1.333333	2.000000
X2	10.000000	4.000000	0.888889
X3	5.000000	1.000000	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	500.000000	INFINITY	140.000000
3	500.000000	INFINITY	200.000000
4	300.000000	INFINITY	300.000000
5	960.000000	160.000000	93.333336
6	3600.000000	280.000000	720.000000

3) For the next what if analysis, I decided to add to the amount of time allowed for production. I increases the amount of time allowed for production from 960 minutes to 1500 minutes per day. Since Wilson manufacturing makes more profit from softballs, you would think that the amount of softballs produced would increase with them having more time to produce them. With all other constraints kept the same, I plugged the new problem into LINDO. The amount of softballs produced increased from 300 dozen to 500 dozen per day, and the amount of baseballs produced decreased from 360 dozen to 120 dozen per day. The surplus of baseballs was 380 dozen for both quantity and time. With the amount of production time increased, Wilson's profits were increased to $5,840. LINDO results:

Maximize 7X1 + 10X2

Subject to

X1 < 500

X2 < 500

X1 + 2X2 < 1500

5X1 + 6X2 < 3600

END

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 5840.000

VARIABLE	VALUE	REDUCED COST
X1	120.000000	0.000000
X2	500.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	380.000000	0.000000
3)	0.000000	1.600000
4)	380.000000	0.000000
5)	0.000000	1.400000

NO. ITERATIONS= 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	7.000000	1.333333	7.000000
X2	10.000000	INFINITY	1.600000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	500.000000	INFINITY	380.000000
3	500.000000	99.999992	316.666656
4	1500.000000	INFINITY	380.000000
5	3600.000000	1900.000000	600.000000

4) The last what if analysis I decided to analyze was the deletion of a variable. I decided to delete the production of baseballs. The profit should decrease because the production of both baseballs and softballs with the current constraints are necessary to maximize profit. With the deletion of baseballs, Wilson's profit from softball production was $5,000 with the production of 500 dozen softballs per day. The surplus of time was 460 minutes and the surplus of cowhide was 600 square feet. If you were to delete softballs instead of baseballs, the profit from baseballs would be $3,500, with a surplus of 460 minutes and 1100 square feet of cowhide. So it would be more profitable to produce only softballs than only baseballs, with fewer surpluses of materials. But it would still be better to produce both baseballs and softballs with the original constraints provided. LINDO Results for both scenario's:

a. Maximize 10X

Subject to

X< 500

X < 960

6X < 3600

END

LP OPTIMUM FOUND AT STEP 1

OBJECTIVE FUNCTION VALUE

1) 5000.000

VARIABLE	VALUE	REDUCED COST
X	500.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	10.000000
3)	460.000000	0.000000
4)	600.000000	0.000000

NO. ITERATIONS= 1

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X	10.000000	INFINITY	10.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE

2	500.000000	100.000000	500.000000
3	960.000000	INFINITY	460.000000
4	3600.000000	INFINITY	600.000000

b. Maximize 7X

Subject to

X< 500

X < 960

5X < 3600

END

LP OPTIMUM FOUND AT STEP 0

OBJECTIVE FUNCTION VALUE

1) 3500.000

VARIABLE	VALUE	REDUCED COST
X	500.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	7.000000

3)	460.000000	0.000000
4)	1100.000000	0.000000

NO. ITERATIONS= 0

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT	ALLOWABLE	ALLOWABLE
	COEF	INCREASE	DECREASE
X	7.000000	INFINITY	7.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT	ALLOWABLE	ALLOWABLE
	RHS	INCREASE	DECREASE
2	500.000000	220.000000	500.000000
3	960.000000	INFINITY	460.000000
4	3600.000000	INFINITY	1100.000000