Detailed Step-by-Step description of the Wilson Problem

Part A. LP Formulation

Step 1. Figure out the problem

Although it may seem obvious, the first step in solving the Wilson Problem is to understand what the problem is. Wilson Manufacturing, a producer of baseballs and softballs, is looking to maximize their profits while staying within the constraints of their limited resources, such as cowhide, for the productions of the balls, production time and the amount of each type of ball produced. So we know that we will need to set variable for each the baseballs and the softballs and we will need to set constraints to show how to maximize profit while proving that the company is staying within their production capacity and resource usage.

Step 2. Define Variables

When graphing the Wilson problem, first we need to define our variables so we can formulate the necessary equations. We will let variable X1 represent the number of baseballs, in dozens, which Wilson will produce and X2 represents the number of softballs, in dozens, which Wilson will produce. Below are the variables shown in equation form:

X1 = number of baseballs in dozens

X2 = number of softballs in dozens

Step 3. Constraints

When settings the constraints for the Wilson problem, we need to see what limited resources are being used and any limits on the number of balls to be produced. The constraints that are given within the problem are that no more than 500 dozen baseballs and no more than 500 dozen softballs can be produced in a day, the total of 3600 sq. ft. of cowhide sheet are available each day and must be used in the production of both types of balls and a total of 960 minutes are available for production each day. The constraints that are assumed in the problem, although not stated, are that Wilson cannot produce a negative number of baseballs or softballs and thus a non-negative constraint must be set while graphing this problem. In addition to the above total constraints, there are individual constraints in the production of baseballs and softballs. Baseballs require 5 sq. ft. of cowhide, to include waste, and 1 minute of production time, whereas softballs require 6 sq. ft. of cowhide and 2 minutes of production time.

Below are the graphing equivalents of the above constraints.

X1 ≤ 500

$X2 \leq 500$

$X1 \geq 0$

$X2 \geq 0$

$5X1 + 6X2 \leq 3600$

$X1 + 2X2 \leq 960$

Step 4. Breakeven or Maximize Profit

The final requirement before graphing the problem is to see what either Wilson or you are trying to get out of, which is normally the breakeven point or figuring out how to maximize their profit given their projected profit per sale of a dozen baseballs and softballs. The Wilson problem gives us the profit of baseballs and softballs, which they project at $7 and $10 respectively. Below is the maximum profit equation that will be used to see how much of each type of ball must be produced to reach the maximum profit:

$7X1 + 10X2$

Step 5. Making inequalities into equality equations

In order to graph the above equations, we need to make them into equalities (equal signs) instead of in inequalities (less than/greater than or equal to). Below are the equalities:

$X1 = 500$

$X2 = 500$

$X1, X2 = 0$

$5X1 + 6X2 = 3600$

$X1 + 2X2 = 960$

Step 6. Finding X and Y intercepts

In order to accurately graph lines for the equations in Step 5, we need to find where the line is going to intercept the X and Y(X2) axis. For each equation, in order to solve for X1, we need to set X2 to 0, and to solve for X2, we need to set X1 to 0. The X1 intercepts would be shown as (X1, 0) and the Y (X2) intercepts would be shown as (0, X2). Below are the steps to reach the X1 and X2 intercepts:

$5X1 + 6X2 = 3600$

X1 intercept

$5X1 + 0 = 3600$

$5X1 \div 5 = 3600 \div 5$

$X1 = 720$

X2 intercept

$0 + 6X2 = 3600$

$6X2 \div 6 = 3600 \div 6$

$X2 = 600$

Intercepts

(720, 0) (0, 600)

$X1 + 2X2 = 960$

X1 intercept

$X1 + 0 = 960$

$X1 = 960$

X2 intercept

$0 + 2X2 = 960$

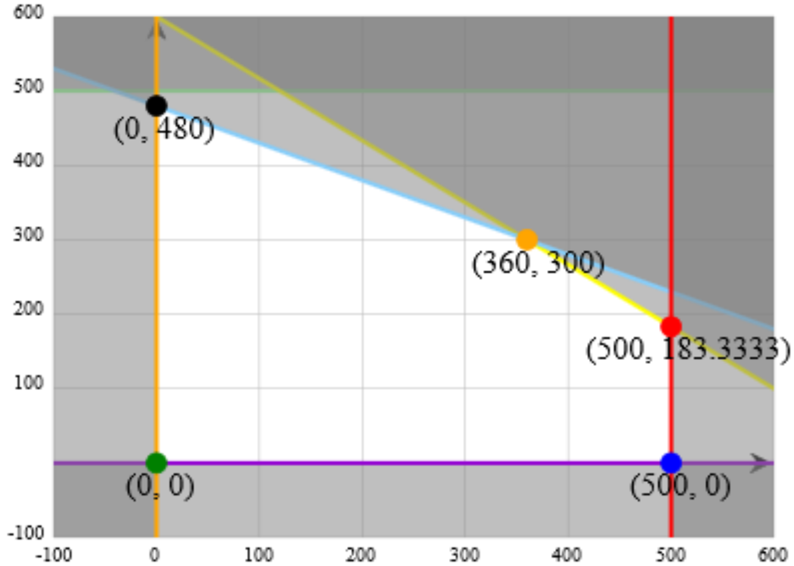
$2X2 \div 2 = 960 \div 2$

$X2 = 480$

Intercepts

(960, 0) (0, 480)

Part B. Graphing



Part C. Suggested Solutions

Wilson is considering manufacturing either 300 of each baseballs and softballs or 350 of each baseballs and softballs and is trying to characterize whether the point is either an interior point, extreme point or infeasible point as well as why neither is an optimal solution.

Step 1. Defining the points

Before we can characterize the solutions, we must first define each type of point, e.g. interior, extreme and infeasible. An interior point is any x-y coordinate that falls within the feasible region, the white area in the graph in part B, and is not one of the corner point. An extreme point is any point that is within the feasible region and is also a corner point. An infeasible point is any point that falls outside of the feasible region, no matter how far outside of the region.

Step 2. Characterizing the solution

The first step in characterizing the solution as either an interior point, extreme point or infeasible point is that you must find the point x-y coordinates on the graph in Part B. For the (300, 300) solution, it is an interior point as it falls within the feasible region, but is not a corner point. For the (350, 350) solution, it is an infeasible point as it falls outside of the feasible region.

Step 3. Non-Optimal Solution Explanation

In order to explain why neither the 300 baseballs and 300 softballs solution nor the 350 baseballs and 350 softballs solution is optimal, we need to go back to the constraints to make sure that the values work and if they do, we will test each constraint to see why it isn't optimal.

Feasible Region Test

The first test to see if either coordinates are feasible, we must check the coordinates on the graph in Part B. The 300 baseballs and 300 softballs solution meets the feasible region test, whereas the 350 baseballs and 350 softballs does not land within the feasible region and is not an optimal solution as Wilson is not capable of manufacturing the amounts required in the suggested solution. In other words:

300 baseballs and 300 softballs – Pass

350 baseballs and 350 softballs – Fail

Now that we see that the 300 of each type of ball solution passes the Feasible Region test, we now have to move onto the constraints test to verify that each of the constraints has been met.

Constraint Test

In order to verify that each of the constraints have been met, we need to put the proposed solutions into each of the constraints. The first test we need to conduction is the production limit test. The production limit constraint was that no more than 500 of each type of ball could be made and that the number made couldn't be negative. In order for this to be true, we will use the following inequality:

$0 \leq X1 \leq 500$

$0 \leq X2 \leq 500$

For the 300 of each type of ball, the inequality would read as:

$0 \leq 300 \leq 500$ – Pass as it is a true statement.

The next constraint we need to test is the production limit constraint $X1 + 2X2 \leq 960$.

$300 + 2(300) \leq 960$

$300 + 600 \leq 960$

$300 + 600 = 900$

$900 \leq 960$ – Pass as 900 is less than 960.

The final constraint is the material (cowhide) constraint ($5X1 + 6X2 \leq 3600$).

$5(300) + 6(300) \leq 3600$

$1500 + 1800 \leq 3600$

$3300 \leq 3600$ – Pass

Now that we know that the 300 of each type of ball has passed both the feasible region and constraint test, we need to explain why it is not the optimal solution. We can do this one of two ways:

1. Show how much production time and materials are left over at the end of each day (waste) or,
2. Find the optimal solution.

For the first way, there were 60 minutes of production time and 300 sq. ft. of material left over after one day of producing 300 of each type of ball. Normally production companies want as little waste as possible as the time/material lost is an opportunity cost. The second way is to find the optimal solution. The way we find the optimal solution is be taking the material and production time equalities, and zeroing out one of the variables. To do this, we take the material equality and subtract the production time equality from it:

$5X1 + 6X2 = 3600$

$- X1 + 2X2 = 960$

We need to make one of the variables cancel the other out, in this case X2, by multiplying the entire bottom equation by 3.

$5X1 + 6X2 = 3600$

$- 3\{X1 + 2X2 = 960\}$

Making it:

$5X1 + 6X2 = 3600$

$- 3X1 + 6X2 = 2880$

The two 6X2s will minus each other out making the new equation:

$2X1 = 720$

$X1 = 360$

Once we know what X1 equals, we pick one of the equations, the production time equation, to find X2:

$360 + 2X2 = 960$

$360 + 2X2 – 360 = 960 – 360$

$2X2 = 600$

$X2 = 300$

Now to compare the two solutions:

300 baseballs and 300 softballs	360 baseballs and 300 softballs
7(300) + 10(300) = MaxP	7(360) + 10(300) = MaxP
2100 + 3000 = MaxP	2520 + 3000 = MaxP
5100 = MaxP	5520 = MaxP

Through the graph above, we can determine that 360 baseballs and 300 softballs is the optimal solution as 300 baseballs and 300 softballs doesn't maximize Wilson's profit nor does the 350 baseballs and 350 softballs fall within the feasible range thus preventing it from being possible solution.

LINDO OUTPUT

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) **5520.000**

VARIABLE	VALUE	REDUCED COST
X1	**360.000000**	0.000000
X2	**300.000000**	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	140.000000	0.000000
3)	200.000000	0.000000
4)	0.000000	1.000000
5)	0.000000	2.000000

NO. ITERATIONS= 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	7.000000	1.333333	2.000000
X2	10.000000	4.000000	1.600000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	500.000000	INFINITY	140.000000
3	500.000000	INFINITY	200.000000
4	3600.000000	280.000000	720.000000
5	960.000000	160.000000	93.333336

Conclusions: The optimal solution 360 baseballs and 300 softballs and the optimal value 5520 agree with the graphical solution. The graphical method is limited to two-dimensional (2-variables) problem, while LINDO is generalization of graphical results that solution is always on of the vertices to larger problems with many decision variables.

The rest of elements in the LINDO report are about sensitivity analysis very useful to the manager.