The Wilson Problem:
Graph is at the end.

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 5520.000

VARIABLE	VALUE	REDUCED COST
X1	360.000000	0.000000
X2	300.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	2.000000
4)	140.000000	0.000000
5)	200.000000	0.000000

NO. ITERATIONS= 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	7.000000	1.333333	2.000000
X2	10.000000	4.000000	1.600000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	3600.000000	280.000000	720.000000
3	960.000000	160.000000	93.333336
4	500.000000	INFINITY	140.000000
5	500.000000	INFINITY	200.000000

Looking at the sensitive analysis of the Wilson Problem, the optimal solution is 5520 where x1 (numbers of baseball) should be 360 and x2 (numbers of softball) should be 300. This is also the same optimal solutions and value as the ones I have in my hand imputation of this problem below.

One of the most important parts of the sensitive analysis is the ranges. As we know the objective function is 7x1 + 10x2 which is also shown in the first two column of “OBJ COFFICIENT RANGES” section. Also in this section are columns called “Allowable decrease” and “Allowable Increase.” These two columns help to identify the ranges in which x1 and x2 (let’s call it c1 and c2) can change while still keeping its optimal solution. For example, for c1 which is 7, the allowable increase is approximately 1.3 and the allowable decrease is 2. So 7 + 1.3 = 8.3 and 7 – 2 = 5; we can say that c1 can change between 5 and 8.3 (5 <= c1 <= 8.3) and the solution can still be optimal. In other words, the price of one baseball can be between 5 and 8.3. Now for c2, 10 + 4 = 14 and 10 – 1.6 = 8.4; the price for one softball can be between 8.4 and 14 (8.4 <= c2 <= 14) and the solution will still be optimal.

The same process is used to find the sensitivity ranges for the constraints (we will call q1, q2, q3, and q4 for each of the constraints respectively). Constraint one is 3,600 so 3600 + 280 = 3880, and 3600 – 720 = 2880. Q1 can change between 2880 and 3880 (2880 <= q1 <= 3880) and the current solution will still remain optimal. Q2 can change between 866.7 and 1120, q3 can be between 360 to infinity (360 <= q3), and q4 can be between 300 to infinity and the solution will still remain optimal.

Row 2 and 3 in the sensitivity analysis represents the cowhide sheet and production time constraints, so under the section on the top of the analysis which says "SLACKS AND SURPLUS" and DUAL PRICES," we can use this information to define the marginal value of one additional unit of resource. Dual price is also called shadow prices that the textbook talks about. So row 2 has a dual price of 1; it means that for every additional cowhide sheet produced, the profit will increase by 1 dollar. And row 3 has a dual price of 2 which means that for every additional minute of production, the profit will increase by 2 dollars. Row 3 and 4 have 0 dual prices so it does not impact those constraints much. However row 3 and 4 do have a slack of 140 and 200 respectively since the optimal solution is 360 and 300. 500-360 = 140 and 500-300 = 200.

<u>THE DARK SIDE OF LINEAR PROGRAMMING</u>

1. Unbounded Feasible region

```
MAX   - 4 X1 - 2 X2
 SUBJECT TO
      2)  X1 >=  4
      3)  X2 <=  2
```

```
LP OPTIMUM FOUND AT STEP      1

     OBJECTIVE FUNCTION VALUE

     1)    -16.00000

 VARIABLE       VALUE        REDUCED COST
     X1        4.000000        0.000000
     X2        0.000000        2.000000


    ROW   SLACK OR SURPLUS    DUAL PRICES
     2)        0.000000       -4.000000
     3)        2.000000        0.000000

 NO. ITERATIONS=      1
```

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	-4.000000	4.000000	INFINITY
X2	-2.000000	2.000000	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	4.000000	INFINITY	4.000000
3	2.000000	INFINITY	2.000000

Managerial interpretation of the sensitivity analysis above:

- The optimal solution of this unbounded feasible region is -16 and the optimal values are 4 and 0 for x1 and x2 respectively.
- Constraint one: for every additional one unit of resource, the profit will decrease by 4 dollars (Dual price = -4)
- Constraint two has a slack value of 2 and reduced cost of 2 dollars.
- Objective coefficient ranges:
  - $c1 <= 0$
  - $c2 <= 0$

This means that the ranges of x1 and x2 can be anywhere from negative infinity to 0 and still keep the solution optimal.

- Constraints ranges:
  - 0 <= q1 <= INFINITY
  - 0 <= q2 <= INFINITY

2. Multiple Optimal Solution

```
MAX     6 X1 + 4 X2
 SUBJECT TO
       2)  X1 + 2 X2 <=   16
       3)  3 X1 + 2 X2 <=   24
 END
```

LP OPTIMUM FOUND AT STEP 1

OBJECTIVE FUNCTION VALUE

1) 48.00000

VARIABLE	VALUE	REDUCED COST
X1	8.000000	0.000000
X2	0.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	8.000000	0.000000
3)	0.000000	2.000000

NO. ITERATIONS= 1

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	6.000000	INFINITY	0.000000
X2	4.000000	0.000000	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	16.000000	INFINITY	8.000000
3	24.000000	24.000000	24.000000

Managerial interpretation of the sensitivity analysis above:

- The optimal solution LINDO gave is 48 and the optimal values are 8 and 0 for x1 and x2 respectively. However, this problem has multiple solutions but one of the weaknesses of LINDO software is that it only gives one solution. For example, another optimal value is 4 and 6 for x1 and x2 respectively.
  - 6 (8) + 4(0) = 48
  - 6(4) + 4(6) = 48
- Constraint 1 has a slack of 8 because (8) + 2(0) <=16
  8 <= 16 (difference of 8)
- Constraint 2 has a dual price of 2 which means that for every additional one unit of resource, the profit will increase by 2 dollars.
- Objective coefficient ranges:
  - 6 <= c1 <= infinity
  - Negative infinity <= c2 <= 4 (c2 <= 4)

  So c1 can be greater than 6 and c2 can be less than 4 and the solution will still remain optimal.
- Constraint ranges:

- 8 <= q1 <= infinity
  0 <= q2 <= 48

3. Infeasible Solution

Max 5X1 + 3X2
Subject to:
4X1 + 2X2 $\leq$ 8
X1 $\geq$ 4
X2 $\geq$ 6

This solution resulted as an error in the LINDO's software because it is an infeasible solution. Either you can change some of the constraints to make it feasible. For example you can change the last two constraints from x1 >= 4 and x2 >=6 to non-negativity constraint. That will get you the solution below with optimal solution of 12 and optimal value of 0 for x1 and 4 for x2.

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 12.00000

VARIABLE	VALUE	REDUCED COST
X1	0.000000	1.000000
X2	4.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.500000
3)	0.000000	0.000000
4)	4.000000	0.000000

NO. ITERATIONS= 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	5.000000	1.000000	INFINITY
X2	3.000000	INFINITY	0.500000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	8.000000	INFINITY	8.000000
3	0.000000	0.000000	INFINITY
4	0.000000	4.000000	INFINITY

4. Multiple solution

MAX 40 X1 + 30 X2
SUBJECT TO
2) X1 + 2 X2 <= 40
3) 4 X1 + 3 X2 <= 120
END

LP OPTIMUM FOUND AT STEP 1

OBJECTIVE FUNCTION VALUE

1) 1200.000

VARIABLE	VALUE	REDUCED COST
X1	30.000000	0.000000
X2	0.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	10.000000	0.000000
3)	0.000000	10.000000

NO. ITERATIONS= 1

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	40.000000	INFINITY	0.000000
X2	30.000000	0.000000	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	40.000000	INFINITY	10.000000
3	120.000000	40.000000	120.000000

Managerial interpretation of the sensitivity analysis above:

- The optimal solution in this analysis is 1200 while the optimal values are 30 and 0 for x1 and x2 respectively. However this problem has multiple solutions but LINDO only gives one solution. Another optimal value in this problem can be 24 and 8.
  - 40(30) + 30(0) = 1200
  - 40(24) + 30(8) = 1200

- In constraint one, there is a slack of 10 resources because 30 + 2(0) <= 40
  30 <= 40 (difference of 10)

- In constraint two, the dual price is 10 which mean that for every additional one unit of resource, there is a profit of 10 dollars.

- Objective coefficient ranges:
  - 40 <= c1 <= infinity
  - Infinity <= c2 <= 30

- Constraint ranges:
  - 30 <= q1 <= infinity
  - 0 <= q2 <= 180

5. Infeasible solution

Max	14x1 - 13x2
St. to:	x1 >= 4
	4x1+2x2 <= 8
	x2 >= 6

This solution is infeasible solution and LINDO only gives error when there is an infeasible solution. To make this a feasible solution, perhaps consider removing the constraints of minimizing x1 and x2 to 4 and 6 and turn them into non-negativity constraints instead?

6. Unbounded region

```
MAX     4 X1 + 3 X2
 SUBJECT TO
       2)   X1 >=   4
       3)   X2 >=   2
 END
```

```
UNBOUNDED VARIABLES ARE:
 SLK    2
      X1

      OBJECTIVE FUNCTION VALUE

      1)     0.9999990E+08

 VARIABLE        VALUE          REDUCED COST
      X1        4.000000          0.000000
      X2        2.000000          0.000000


     ROW   SLACK OR SURPLUS     DUAL PRICES
      2)         0.000000        -1.000000
      3)         0.000000         0.000000

 NO. ITERATIONS=       2
```

The solution is unbounded so there is no specific optimal solution in this problem which is the reason why LINDO resulted in an error while implementing this problem in the software. To make it feasible, consider either changing the objective function from positive to negative or changing the inequality signs of the constraints

Even though LINDO reported an error, it still gave a small analysis of the problem. A possible optimal solution can be 0.9999990E+08 while the optimal values can be 4 and 2 for x1 and x2 respectively. The dual price or the shadow price for constraint one is -1.

LINDO reported “UNBOUNDED VARIABLES ARE: SLK 2 x1.” This means that the problem is unbounded because of the first constraint and to make it feasible, considers changing that constraint’s directions. When this is done (changed >= to <=) the optimal value is 20 with the solution being x1 = 4 and x2 = 2.

Linear Programming Formulation and Solution

Example 1: Candy Manufacturer

```
 MAX     2 X1 + 1.25 X2
 SUBJECT TO
        2)   0.5 X1 + 0.33 X2 <=   130
        3)   0.5 X1 + 0.67 X2 <=   170
 END

 LP OPTIMUM FOUND AT STEP      1

      OBJECTIVE FUNCTION VALUE
```

1) 520.0000

VARIABLE	VALUE	REDUCED COST
X1	260.000000	0.000000
X2	0.000000	0.070000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	4.000000
3)	40.000000	0.000000

NO. ITERATIONS= 1

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	2.000000	INFINITY	0.106061
X2	1.250000	0.070000	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	130.000000	40.000000	130.000000
3	170.000000	INFINITY	40.000000

Managerial interpretation of the sensitivity analysis above:

- The optimal solution in this problem is 520, and the optimal value is 260 pounds of mixture of half cherries and half mints, and 0 pounds of mixture which is one-third cherries and two-thirds mints.
- Constraint one has a dual price of $4 which mean that for every pound of cherries used in this mixture, there is a profit of $4.
- Constraint two has a slack of 40 because 0.5(260) + 0.67(0) <= 170
  130 <= 70 (difference of 40)
- Objective Coefficient ranges:
  - 1.89 <= c1 <= infinity
  - C2 <= 1.32
- Constraints ranges
  - 0 <= q1 <= 170
  - 130 <= q2 <= infinity

- The candy manufacturer attains maximum sales of \$520 when he produces 260 pounds of mixture A and none of mixture

Example 2: Recycling center

```
MIN     40 X1 + 50 X2
SUBJECT TO
       2)  140 X1 + 100 X2 >=   1540
       3)  60 X1 + 180 X2 >=   140
       4)  X1 >=   0
       5)  X2 >=   0
END
```

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 440.0000

VARIABLE	VALUE	REDUCED COST
X1	11.000000	0.000000
X2	0.000000	21.428572

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-0.285714
3)	520.000000	0.000000
4)	11.000000	0.000000
5)	0.000000	0.000000

NO. ITERATIONS= 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	40.000000	29.999996	40.000000
X2	50.000000	INFINITY	21.428570

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	1540.000000	INFINITY	1213.333374
3	140.000000	520.000000	INFINITY

4	0.000000	11.000000	INFINITY
5	0.000000	0.000000	INFINITY

Managerial interpretation of the sensitivity analysis above:

- The optimal solution of this problem is 440 while the optimal values are x1 = 11 and x2 = 0. (for some reason I received a different optimal solution than the one on the document on Sakai)
- Constraint one has a dual price of $ -0.29 which means that for every additional pound of glass deposited in the recycling center, there is a loss of 29 cents.
- Constraint two has a slack value of 520 and constraint three has a slack value of 11.
- Objective coefficient ranges:
  - 0 <= c1 <= 70
  - 28.6 <= c2 <= infinity
- Constraint ranges:
  - 326.7 <= q1 <= infinity
  - Q2 <= 660
  - Q3 <= 11
  - Q4 <= 0

So, the minimal cost is $440 and it is attainted when Center 1 is open for about 7 days a week (eleven-hours a day) and Center 2 is not open at all.

Example 3: Toques and mitts

```
MAX    2 X1 + 5 X2
SUBJECT TO
     2)  X1 <=   150
     3)  X2 <=   120
     4)  X1 + X2 <=   200
END
```

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 760.0000

VARIABLE	VALUE	REDUCED COST
X1	80.000000	0.000000

X2	120.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	70.000000	0.000000
3)	0.000000	3.000000
4)	0.000000	2.000000

NO. ITERATIONS= 2

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	2.000000	3.000000	2.000000
X2	5.000000	INFINITY	3.000000

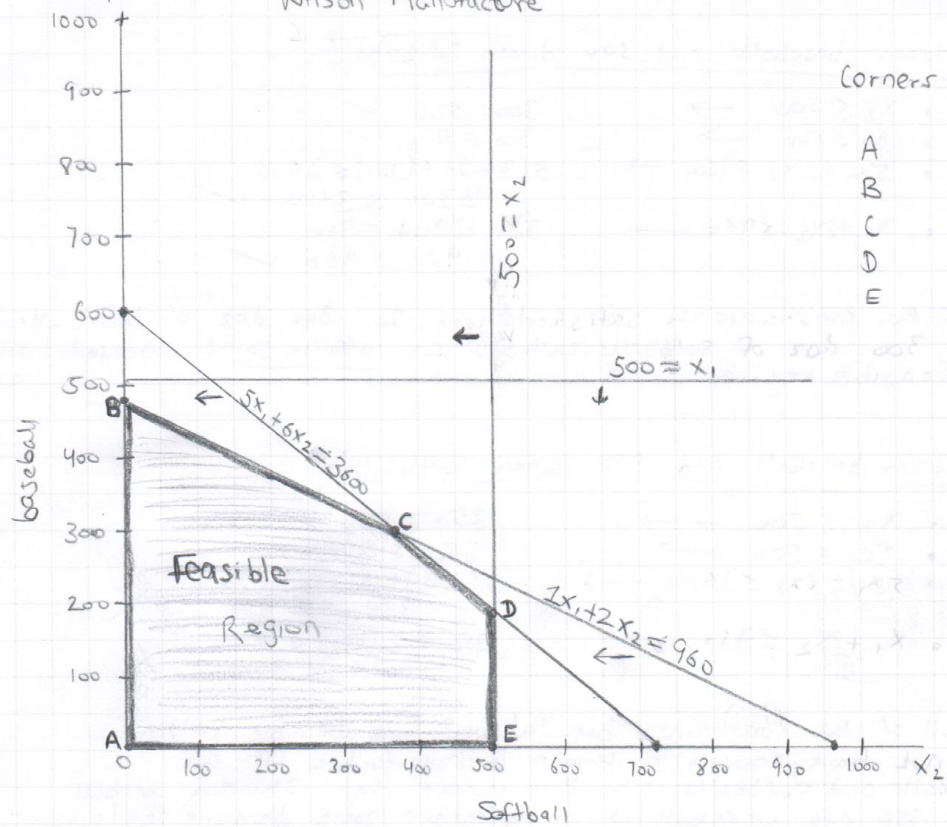
RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	150.000000	INFINITY	70.000000
3	120.000000	80.000000	70.000000
4	200.000000	70.000000	80.000000

Managerial interpretation of the sensitivity analysis above:

- The optimal solution is 760 and the optimal value is 80 torques and 120 mitts.
- Constraint one has a slack of 70 torques because 80 <= 150 (difference of 70).
- Constraint two has a dual price of $3 and constraint three has a dual price of $2
- Objective coefficient ranges:
  - $0 \le c_1 \le 5$
  - $2 \le c_2 \le$ infinity
- Constraint ranges:
  - $80 \le q_1 \le$ infinity
  - $50 \le q_2 \le 200$
  - $120 \le q_3 \le 270$
- The sewing students (and teachers) must make 80 toques and 120 pairs of mitts each week in order to make the most money.

Wilson Manufacture



Corners	X_1	X_2
A	0	0
B	0	480
C	360	300
D	500	183
E	500	0

The Mathematical Model

X_1 = numbers of baseball produced daily
X_2 = numbers of Softball produced daily

$5X_1 + 6X_2 \leq 3600$

X_1	X_2
0	600
720	0

$1X_1 + 2X_2 \leq 9$

X_1	X_2
0	48
960	0

Subject to:

Max $= 7X_1 + 10X_2$ (objective function)

S.T.
$500 \geq X_1$ (Production limits of baseball each day)
$500 \geq X_2$ (Production limits of Softball each day)
$5X_1 + 6X_2 \leq 3600$ (Cowhide Sheet)
$1X_1 + 2X_2 \leq 960$ (production time, in minutes)
$X_1, X_2 \geq 0$ (Non-negativity)

$7X_1 + 10X_2$

A $7(0) + 10(0) = 0$
B $7(0) + 10(480) = 4800$
C $7(360) + 10(300) = 5520$ Maximize (360, 300)
d $7(500) + 10(183) = 3518$
e $7(500) + 10(0) = 3500$