The Wilson Problem:

Max 7x1 + 10x2
S.T.
5x1 + 6x2 <= 3600
1x1 + 2x2 <= 960
x1 <= 500
x2 <= 500

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) 5520.000

VARIABLE	VALUE	REDUCED COST
X1	360.000000	0.000000
X2	300.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	2.000000
4)	140.000000	0.000000
5)	200.000000	0.000000

NO. ITERATIONS= 2

Managerial Interpretations: The optimal solution for the primal problem is X1 = 360, X2 = 300, with optimal values of $5520.
Dual Prices column indicating that they are the optimal solution for the **Dual** problem, which means they are, shadow **Prices** (of each RHS). U1 = 1, U2 = 2, U3 = 0, U4 = 0.

RANGES IN WHICH THE BASIS IS UNCHANGED:

Managerial Interpretations: This part provides the current coefficients values and the range for each that the change in each coefficient for which the optimal solution remain the same.

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
X1	7.000000	1.333333	2.000000
X2	10.000000	4.000000	1.600000

RIGHTHAND SIDE RANGES

Managerial Interpretations: This part provides the current RHS of constraints values and the range for each that the change in each RHS for which the solution to the dual problem (the shadow prices) remain the same.

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	3600.000000	280.000000	720.000000
3	960.000000	160.000000	93.333336
4	500.000000	INFINITY	140.000000
5	500.000000	INFINITY	200.000000

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ROW	(BASIS)	X1	X2	SLK 2	SLK 3	SLK 4
1	ART	0.000	0.000	1.000	2.000	0.000
2	X1	1.000	0.000	0.500	-1.500	0.000
3	X2	0.000	1.000	-0.250	1.250	0.000
4	SLK 4	0.000	0.000	-0.500	1.500	1.000
5	SLK 5	0.000	0.000	0.250	-1.250	0.000

ROW	SLK 5	
1	0.000	5520.000
2	0.000	360.000
3	0.000	300.000
4	0.000	140.000
5	1.000	200.000

Looking at the sensitive analysis of the Wilson Problem, the optimal solution is 5520 where x1 (numbers of baseball) should be 360 and x2 (numbers of softball) should be 300.

One of the most important parts of the sensitive analysis is the ranges. As we know the objective function is 7x1 + 10x2 which is also shown in the first two column of "OBJ COFFICIENT RANGES" section. Also in this section are columns called "Allowable decrease" and "Allowable Increase." These two columns help to identify the ranges in which coefficient of x1 and x2 (let's call it c1 and c2) can change while still keeping its optimal solution. For example, for c1 which is 7, the allowable increase is 1.3 and the allowable decrease is 2. So 7 + 1.3 = 8.3 and 7 – 2 = 5; we can say that c1 can change between 5 and 8.3 (5 <= c1 <= 8.3) and the current optimal solution can still be optimal. In other words, the price of one baseball can be between 5 and 8.3. Now for c2, 10 + 4 = 14 and 10 – 1.6 = 8.4; the price for one softball can be between 8.4 and 14 (8.4 <= c2 <= 14) and the optimal solution will still be optimal.

The same process is used to find the sensitivity ranges for the RHS of constraints (we will call q1, q2, q3, and q4 for each of the constraints respectively). Constraint one is 3,600 so 3600 + 280 = 3880, and 3600 – 720 = 2880. Q1 can change between 2880 and 3880 (2880 <= q1 <= 3880) and the current optimal solution to the Dual problem will still remain optimal. Q2 can change between 866.7 and 1120, q3 can be between 360 to infinity (360 <= q3), and q4 can be between 300 to infinity and the solution will still remain optimal.

Row 2 and 3 in the sensitivity analysis represents the cowhide sheet and production time constraints, so under the section on the top of the analysis which says "SLACKS AND SURPLUS" and DUAL PRICES," we can use this information to define the marginal value of one additional unit of resource. Dual price is also called shadow prices that the textbook talks about. So row 2 has a dual price of 1; it means that for every additional cowhide sheet produced, the profit will increase by 1 dollar. The shadow price will remain the same as long as the cowhide constraint stays within the range of 2,880 and 3,880. And row 3 has a dual price of 2 which means that for every additional minute of production, the profit will increase by 2 dollars. The shadow price will stay the same as long as values stay between the range 866.7 and 1,120. Dual price values are also the optimal values in the Wilson dual problem. Row 3 and 4 have 0 dual prices so it does not impact those constraints within their limits (i.e., range). However row 3 and 4 do have a slack of 140 and 200 respectively since the optimal solution is 360 and 300. 500-360=140 and 500-300=200. Since they have non-negative slack therefore the value of these leftover is zero, that is why there shadow prices U3 and U4 are zero.

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LP OPTIMUM FOUND AT STEP      0

       OBJECTIVE FUNCTION VALUE

       1)      5720.000

 VARIABLE        VALUE          REDUCED COST
       X1        460.000000          0.000000
       X2        250.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          1.000000
       3)         0.000000          2.000000
       4)        40.000000          0.000000
       5)       250.000000          0.000000

NO. ITERATIONS=       0


RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
      X1        7.000000         1.333333         2.000000
      X2       10.000000         4.000000         1.600000

                           RIGHTHAND SIDE RANGES
     ROW         CURRENT        ALLOWABLE        ALLOWABLE
                   RHS           INCREASE         DECREASE
       2     3800.000000        80.000000       920.000000
       3      960.000000       200.000000        26.666666
       4      500.000000        INFINITY         40.000000
       5      500.000000        INFINITY        250.000000
```

I increased RHS of Q1 (cowhide) from 3600 to 3800 (which is within the RHS range) too see how it impacted the dual price and the optimal solution. The dual price stayed the same as I stated above and the optimal solution increased by 200 because I increased cowhides from 3600 to 3800, difference of 200 (dual price of 1 x 200 = 200).

```
LP OPTIMUM FOUND AT STEP      0

       OBJECTIVE FUNCTION VALUE

       1)      5720.000

 VARIABLE        VALUE          REDUCED COST
       X1       210.000000          0.000000
       X2       425.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          1.000000
       3)         0.000000          2.000000
       4)       290.000000          0.000000
       5)        75.000000          0.000000

NO. ITERATIONS=       0


RANGES IN WHICH THE BASIS IS UNCHANGED:

                          OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
      X1        7.000000         1.333333         2.000000
      X2       10.000000         4.000000         1.600000

                          RIGHTHAND SIDE RANGES
     ROW         CURRENT        ALLOWABLE        ALLOWABLE
                   RHS           INCREASE         DECREASE
       2     3600.000000       580.000000       300.000000
       3     1060.000000        60.000000       193.333328
       4      500.000000         INFINITY       290.000000
       5      500.000000         INFINITY        75.000000
```

Similar change occurred in this also; **I change the RHS of the production time from 960 minutes to 1060 minutes**, an increase of 100. As we know the dual price of Q2 is 2 (2 x 100 = 200), hence the reason behind an increase of the optimal solution from 5520 to 5720 (difference of 200).

WILSON DUAL PROBLEM

Minimize 3600U1 + 960U2 + 500U3 + 500U4
S.T.
5U1 + 1U2 + 1U3 >= 7
6U1 + 2U2 + 1U4 >= 10
U1, U2, U3, U4 >= 0

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```
LP OPTIMUM FOUND AT STEP      3

       OBJECTIVE FUNCTION VALUE

       1)      5520.000

 VARIABLE        VALUE          REDUCED COST
       U1         1.000000          0.000000
       U2         2.000000          0.000000
       U3         0.000000        140.000000
       U4         0.000000        200.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000       -360.000000
       3)         0.000000       -300.000000

NO. ITERATIONS=       3


RANGES IN WHICH THE BASIS IS UNCHANGED:

                          OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                  COEF          INCREASE         DECREASE
      U1      3600.000000       280.000000       720.000000
      U2       960.000000       160.000000        93.333336
      U3       500.000000         INFINITY       140.000000
      U4       500.000000         INFINITY       200.000000

                          RIGHTHAND SIDE RANGES
     ROW         CURRENT        ALLOWABLE        ALLOWABLE
                   RHS          INCREASE         DECREASE
       2        7.000000         1.333333         2.000000
       3       10.000000         4.000000         1.600000
```

The optimal solution for this dual problem is 5520 which is the same as the optimal solution of the original Wilson problem, making means they are in economic equilibrium. Optimal values of U1= 1 and U2=2 which were the dual prices in the original Wilson problem. The same is true for vice versa, the dual price of the dual problem is optimal values of the original Wilson problem, -360 and -300 (except the dual price is negative). The sensitivity ranges for U1 is 3600-720 = 2880 and 3600 + 280 = 3880; so as long as cowhides are between 2880 and 3880, we can infer that for every additional cowhide produced, the profit will increase by $1. Sensitivity ranges for U2 is 960 + 160 = 1120 and 960 – 93.3 = 866.7; so as long as the production time is between 866.7 and 1120, we can infer that for every additional time, the profit will increase by $2. Sensitivity range for U3 is between 360 and infinity and U4 is between 300 and infinity. There is a “reduced cost” or unused baseballs and softball of 140 and 200 respectively. The minimum range of U3 and U4 is 360 and 300 which is the optimal value of the original Wilson problem. The RHS analysis of constraint one is between 5 and 8.3 and for constraint two is between 8.4 and 14.

What-if analysis

DELETE A CONSTRAINT

Delete the cowhide constraint

Since cowhide constraint has a slack of 0 that means it is a binding constraint which means that the optimal solution will be affected. The optimal solution changed from 5520 to 5800 and the optimal values changed to 500 and 230 for X1 and X2 respectively. In this analysis, baseball materials are fully used while softballs have a slack of 270. The dual price is 5 for time production (row 2) which means that for every additional minute, the profit will increase by $5 and for every additional production of baseballs, there is a profit of $2. Sensitivity ranges for objective coefficient ranges are between 5<= c1<= infinity and 0 <= c2 <= 14. The sensitivity ranges for RHS ranges are 500 <= Q1 <= 1500, 0 <= Q2 <= 960, and 230 <= Q3 <= infinity.

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```
LP OPTIMUM FOUND AT STEP      2

       OBJECTIVE FUNCTION VALUE

       1)      5800.000

 VARIABLE        VALUE          REDUCED COST
       X1        500.000000          0.000000
       X2        230.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          5.000000
       3)         0.000000          2.000000
       4)       270.000000          0.000000

NO. ITERATIONS=       2

RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                  COEF          INCREASE         DECREASE
      X1        7.000000        INFINITY          2.000000
      X2       10.000000        4.000000         10.000000

                           RIGHTHAND SIDE RANGES
     ROW         CURRENT        ALLOWABLE        ALLOWABLE
                  RHS           INCREASE         DECREASE
       2      960.000000      540.000000       460.000000
       3      500.000000      460.000000       500.000000
       4      500.000000        INFINITY       270.000000
```

Delete the X1<500 constraint

Since the slack value of X1 constraint is 140, meaning that this constraint is not binding, i.e., is not important, therefore deleting does not have any impact on the optimal solution. The optimal solution will not be affected as shown in Lindo output, because the optimal solution is still 5520 in this analysis. Along with that, the optimal values stayed the same too, X1 = 360 and x2= 300 and the dual price also stayed the same for constraint 1 and constraint 2 ($1 and $2 respectively).

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```
 LP OPTIMUM FOUND AT STEP       2

        OBJECTIVE FUNCTION VALUE

        1)       5520.000

  VARIABLE        VALUE          REDUCED COST
        X1        360.000000          0.000000
        X2        300.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)          0.000000          1.000000
        3)          0.000000          2.000000
        4)        200.000000          0.000000

 NO. ITERATIONS=        2

 RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
 VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
       X1        7.000000         1.333333         2.000000
       X2       10.000000         4.000000         1.600000

                           RIGHTHAND SIDE RANGES
      ROW         CURRENT        ALLOWABLE        ALLOWABLE
                    RHS          INCREASE         DECREASE
        2     3600.000000      1200.000000       720.000000
        3      960.000000       160.000000       240.000000
        4      500.000000         INFINITY       200.000000
```

Addition of a variable

Let's say Wilson manufacturer produces tennis balls along with baseballs and softball and it takes 3 minutes of production time, requires 2 square feet of cowhides, and they can only produce a maximum of 500 tennis balls per day. The profit they will generate from this tennis balls will be 15 dollars because tennis balls are cheaper to make. Before running the Lindo package, we should ask is it profitable to produce the new product? What goes in a unit of new product 2 (U1) + 3(U2) + 0(U3) + 0(U4) = 2(1) + 3(2) = $8, since the profit $15 is more than its cost $8, we will produce the product, but how much? For this we have to run lindo.

The new mathematical model will look as follows:

Max 7X1 + 10X2 + 15X3 (objective function)
S.T.
5X1 + 6X2 + 2X3 <= 3600 (cowhide production)
1X1 + 2X2 + 3X3 <= 960 (production time)
X1 <= 500 (production of baseballs)
X2 <= 500 (production of softballs)
X3 <= 500 (production of tennis balls)

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Reports Window

LP OPTIMUM FOUND AT STEP      1

       OBJECTIVE FUNCTION VALUE

       1)      5800.000

 VARIABLE        VALUE          REDUCED COST
       X1       500.000000          0.000000
       X2       170.000000          0.000000
       X3        40.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          0.000000
       3)         0.000000          5.000000
       4)         0.000000          2.000000
       5)       330.000000          0.000000
       6)       460.000000          0.000000

NO. ITERATIONS=       1

RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                   COEF          INCREASE         DECREASE
      X1        7.000000         INFINITY         2.000000
      X2       10.000000         2.153846         0.000000
      X3       15.000000         0.000000         7.000000

                           RIGHTHAND SIDE RANGES
     ROW         CURRENT        ALLOWABLE        ALLOWABLE
                   RHS           INCREASE         DECREASE
       2     3600.000000       280.000000       793.333313
       3      960.000000      1073.333374        93.333336
       4      500.000000       183.076935       140.000000
       5      500.000000         INFINITY       330.000000
       6      500.000000         INFINITY       460.000000
```

So by adding one more variable, the optimal profit increased to 5800 from 5520, an increase of 280. The optimal values are to produce 500 baseballs, 170 softballs, and 40 tennis balls. Adding this new product with cause a slack of 330 softballs and 460 tennis balls that could've been produced. It will also have a dual price of $5 for the production time and a dual price of $2 for the production of baseballs. This new addition of variable will allow the cowhide constraint range to be between 2807 and 3880 (manufacturers can use a little less cowhides than the original Wilson problem and still maintain their optimal profit) and allow the production time to be between 866.7 and 2033.3 and still remain optimal. The new product seems profitable looking at the new optimal solution and the broader objective coefficient and RHS ranges so it would be profitable to add the new resource in the Wilson productions.

Addition of a constraint

Along with cowhides and production time, Wilson manufacturer have to add in an extra resource to make baseballs and softballs; they have to add rubber in their productions. They need 3 square feet of rubber for baseball and 2 square feet of rubber for softball and there can be a maximum of 1500 square feet of rubber used per day. So the new constraint would be as follows:

3x1 + 2x2 <= 1500

If you plug in the values of x1=360 and x2=300 in this new constraint, you will get—3(360) + 2(300) = 1680. Since this number is greater than 1500, the new constraint is not stratified, therefore will affect the optimal solution (as shown below). The optimal solution increased to 5340 and the optimal values changed to 270 and 345 for x1 and x2 respectively. In this solution, there will be a slack of 180 cowhides, 230 baseballs, and 155 softballs.

Reports Window

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LP OPTIMUM FOUND AT STEP      2

       OBJECTIVE FUNCTION VALUE

       1)      5340.000

 VARIABLE        VALUE          REDUCED COST
       X1       270.000000          0.000000
       X2       345.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)       180.000000          0.000000
       3)         0.000000          4.000000
       4)         0.000000          1.000000
       5)       230.000000          0.000000
       6)       155.000000          0.000000

NO. ITERATIONS=       2


RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE         ALLOWABLE
                  COEF          INCREASE          DECREASE
      X1        7.000000        8.000000          2.000000
      X2       10.000000        4.000000          5.333333

                           RIGHTHAND SIDE RANGES
     ROW         CURRENT        ALLOWABLE         ALLOWABLE
                  RHS           INCREASE          DECREASE
       2     3600.000000        INFINITY        180.000000
       3      960.000000       90.000000        460.000000
       4     1500.000000      180.000000        540.000000
       5      500.000000        INFINITY        230.000000
       6      500.000000        INFINITY        155.000000
```

Deletion of a variable

I decided to delete X2, the softball variable, so see what impact it has on the LINDO's analysis. Since I am deleting X2, I must also delete the X2 <= 500 constraint and all of the other x2 variables in the constraints.

My new mathematical model will look as follows:
Max 7x1
S.T.
5x1 <= 3600
1x1 <= 960
x1 <= 500

```
LP OPTIMUM FOUND AT STEP      1

       OBJECTIVE FUNCTION VALUE

       1)      3500.000

 VARIABLE        VALUE          REDUCED COST
       X1       500.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)      1100.000000          0.000000
       3)       460.000000          0.000000
       4)         0.000000          7.000000

NO. ITERATIONS=       1


RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                  COEF          INCREASE         DECREASE
      X1        7.000000        INFINITY         7.000000

                           RIGHTHAND SIDE RANGES
     ROW         CURRENT        ALLOWABLE        ALLOWABLE
                  RHS           INCREASE         DECREASE
       2      3600.000000       INFINITY      1100.000000
       3       960.000000       INFINITY       460.000000
       4       500.000000      220.000000      500.000000
```

The optimal value decrease from 5520 to 3500 (decrease of 2020, because of not producing a profitable product!) and the optimal value is 500= x1. There will be a slack of 1100 cowhides, and 460 minutes of production. The slack are increasing because of not using them. And there will also a dual price of $7 for every additional baseball produced. The RHS range for cowhides is between 2500 to infinity, for time production is between 500 to infinity, and for baseball production is between 0 to 720. The objective coefficient range for X1 is between 0 to infinity. Deleting a variable will have a huge impact on the optimal solution!