Confidence Interval for Population Variance σ^2

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha/2}}$$

Let $\alpha = .05$

$X \sim N$

$\sigma^2 = ?$

$n = 10$

$S = 4.93$

$$\frac{9(4.93)^2}{19.023} \le \sigma^2 \le \frac{9(4.93)^2}{2.70}$$

$$P[11.5 \le \sigma^2 \le 81.10] \geq .95$$

$$P[3.39 \le \sigma \le 9.01] \geq .95$$

For large sample size n

$X \sim N$

$\sigma = ?$

$n = 500$

$S = 4.93$ (say)

$$\frac{S}{1 + Z_{\alpha/2}/\sqrt{2n}} \le \sigma \le \frac{S}{1 - Z_{\alpha/2}/\sqrt{2n}}$$

$$\frac{4.93}{1 + 1.96(.0316)} \le \sigma \le \frac{4.93}{1 - 1.96(.0316)}$$

$$\frac{4.93}{1.0625} \le \sigma \le \frac{4.93}{.9375}, \quad P[4.64 \le \sigma \le 5.26] \geq .95$$

$$P[21.53 \le \sigma^2 \le 27.67] \geq .95$$

The chance That This interval Contains The population Variance is <u>at least</u> 95%.

Test of hypothesis concerning population Variance σ^2

$H_0: \sigma_0^2 = 2$ The statistic $\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$

$H_a: \sigma_0^2 \neq 2$

$n = 16, s^2 = 2.22$, say $\alpha = .05$

The computed statistic is $\chi_0^2 = 16.65$

However The critical values are $\chi^2_{15,.025} = 27.488$

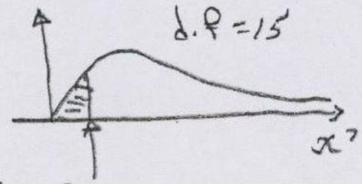
and $\chi^2_{15,.975} = 6.262$

Therefore, there is no evidence to reject H_0

$H_0: \sigma_0^2 = 2$

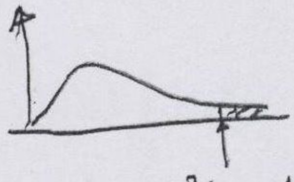
$H_a: \sigma_0^2 < 2$

with $n = 16, s^2 = 2.22, \alpha = .05$



observe statistic $\frac{15(2.22)}{2} = 16.65$ $\chi^2 = 7.261$

There is no evidence to reject H_0



$\chi^2 = 24.996$

$H_0: \sigma_0^2 = 2$

$H_a: \sigma_0^2 < 2$

with $n = 16, s^2 = 2.22, \alpha = .05$

computed statistic is $\frac{15(2.22)}{2} = 16.65$

Therefore There is no evidence to reject H_0

Determination of α 1% B 5% B 10% B

Type I error 99% G 95% G 90% G

Consumer risk