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## Access to LINDO in UB Computer Labs:

*Click on:*

<https://citrix.ubalt.edu>

Click on “INSTALL” while check “I agree box”

It ask for your UB ID

“Installation is completed successfully”

Now you should have access to LIND.

**1. Setting up a sample model to be solved by LINDO** (for ease in editing you should work in word.doc environment, not LINDO yet)

*An illustrative numerical example:* The carpenter's problem deals with finding out how many Tables XT and how many Chairs XC tables XT to make per week. His net profits for one XT and one XC are \$5 and \$3, respectively. Therefore his **objective** is to maximize his net total profit:

Maximize  $5XT + 3XC$

That is, the net income (say, in dollars, or tens of dollars) from selling XT units of tables and XC units of chairs. This is subject to constraining factors that usually *come from*

*outside*, are the limitations on labors (this limitation comes from his family) and raw material resources (this limitation comes from scheduled delivery).

Production times required for a table and a chair are measured at different times of day, and estimated to be 2 hours and 1 hour, respectively. Total labor hours per week are only 40 hrs. Raw materials required for a table and a chair are 1, and 2 units respectively. Total supply of raw material is 50 units per week. Therefore, the LP formulation model is:

$$\text{Maximize } 5XT + 3XC$$

*Subject to:*

$$2XT + XC \leq 40 \text{ labor constraint}$$

$$XT + 2XC \leq 50 \text{ material constraint}$$

*and both XT, XC are non-negative, i.e.  $XT \geq 0$ ,  $XC \geq 0$ .*

This is a **linear** (because all variable have power of 1) **programming** (not a Computer programming language), but its best (optimal) solution (the pair of unknown XT, XC) programs the carpenter activity for a week. We refer to this model the **Carpenter Decision Problem**.

Now we will use LIND to find the optimal solution for the Decision Variable (XT, XC).

**2. How to prepare the model to be solved by LINDO?** (For ease in editing you should work in word.doc environment, not LINDO yet)

*We now discuss how to prepare the carpenter model for LINDO software.*

To solve the following model (by LINDO):

$$\text{Maximize } 5XT + 3XC$$

*Subject to:*

$$2XT + XC \leq 40 \text{ labor constraint}$$

$$XT + 2XC \leq 50 \text{ material constraint}$$

*and both XT, XC are non-negative i.e.  $XT \geq 0$ , and  $XC \geq 0$ .*

First *format this model for LINDO*. It will be almost the same with minor Editing:

$$\text{Max } 5XT + 3XC$$

S. T.

$$2XT + XC < 40$$

$$XT + 2 XC < 50$$

End

- In general, an optimization model will consist of the following three (i.e. rows) items:
  - Row 1: **Objective Function**: The objective function is a formula that expresses exactly what it is you want to optimize.
  - Other Rows are the **Constraints**: Almost without exception, there will be some limit on the values the variables in a model can assume--at least one resource will be limited (e.g., time, raw materials, your department's budget, etc.).
  - **Variables** (or decision variables): These are the quantities you have under your control. You must decide what the best values of the variables are. The goal of optimization is to find the values of a model's variables that generate the best value for the objective function, subject to any limiting conditions placed on the variables.
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- LINDO interprets the "<" symbol as less-than-or-equal-to rather than strictly-less-than. Similarly the ">" symbol as greater than-or-equal-to rather than strictly-less-than.
  - The objective function should not contain any constant. For example, Max 5XT + 3XC + 100 is not allowed. You add 100 to the profit later when you get LINDO solution.
  - All variables must appear on the left-hand side of the constraints, while numerical values must appear on the right-hand side of the constraints.
  - All variables are assumed to be nonnegative. Therefore, *do not enter* the non-negativity constraints.

### 3. How to Enter the (input) Model to LINDO? [Windows]

Copy this LINDO formatted model (from the Word.Doc) then launch the LINDO package, under its toolbar File button select "New", it will pop-up a new blank window, then past what you copied in there. [Notice: Remove any "?" in the model, after pasting, there should not be any]

*As a related question you may ask: How to run a problem that I have at an earlier session you saved?* If you want to run a problem that you have already save. Under LINDO toolbar File button select "Open", then Table II pop-up, select the problem that you have saved therein

Now, select "Solve" on the menu bar and then click on "Solve".

LINDO will ask you "Do Range (Sensitivity) Analysis?" in a dialog box. Click on "Yes" on the dialog box. The solution will be displayed in a separate "Reports Window". **The output is:**

LP OPTIMUM FOUND AT STEP 2

OBJECTIVE FUNCTION VALUE

1) **110.0000**

| VARIABLE | VALUE            | REDUCED COST |
|----------|------------------|--------------|
| XT       | <b>10.000000</b> | 0.000000     |
| XC       | <b>20.000000</b> | 0.000000     |

| ROW | SLACK OR SURPLUS | DUAL PRICES |
|-----|------------------|-------------|
| 2)  | 0.000000         | 2.333333    |
| 3)  | .000000          | 0.333333    |

NO. ITERATIONS= 2

RANGES IN WHICH THE BASIS IS UNCHANGED

| VARIABLE | OBJ COEFFICIENT RANGES ( |                    |                    |
|----------|--------------------------|--------------------|--------------------|
|          | CURRENT COEF             | ALLOWABLE INCREASE | ALLOWABLE DECREASE |
| XT       | 5.000000                 | 1.000000           | 3.500000           |
| XC       | 3.000000                 | 7.000000           | 0.500000           |

| ROW | RIGHTHAND SIDE RANGES |                    |                    |
|-----|-----------------------|--------------------|--------------------|
|     | CURRENT RHS           | ALLOWABLE INCREASE | ALLOWABLE DECREASE |
| 2   | 40.000000             | 60.000000          | 15.000000          |
| 3   | 50.000000             | 30.000000          | 30.000000          |

Thus, the optimal solution for the problem is:  $XT = 10$  and  $XC = 20$ , with a corresponding optimal objective-function value of **\$110**. Interpretation of the remaining information in the output will be discussed in the classroom.

It is interesting to note that while the net profit for table (\$5) is greater than chair (\$3), quite intuitive one may say that XT should be more than XC to make more profit. Well even for this simple model **intuition does not work**. One needs software like LINDO to find the correct strategy.

For **How To Interpret** the rest of results, visit

<http://home.ubalt.edu/ntsbarsh/opre640c/partX.htm>

For **Modeling Validations**, visit

<http://home.ubalt.edu/ntsbarsh/Business-stat/opre/partv.htm>

#### 4. How do I Save and Print my work or results [Windows]?

You may print any window, including both model and report windows in Word.Doc, use the *Cut* and *Paste*. The Save the Word.Doc You must import them in Word.Doc for necessary analysis

It is preferable as you see to export these two files to a Word.Doc file, say **CarpenterDecision.doc**, to add any analysis, managerial interpretations, etc. Now you have done with LINDO. Your analysis, managerial interpretations, etc can be started. Do not print and hand in **CarpenterDecision.doc**, to anyone (e.g., your Professor) without writing-up completely what every word, number means in Managerial Setting (you will learn this one of your course, e.g. OPRE315 topics).

#### 5. How do solve Unrestricted (i.e. non-standard form) LP?

Suppose you wish to solve the following LP model:

Maximize  $X_1$

Subject To:

$$X_1 + X_2 \geq 0$$

$$2X_1 + X_2 \geq 2$$

$$X_1 \geq 0$$

$$X_2 \leq 0$$

Notice that the feasible region is in the fourth quadrant of Cartesian system.

The LINDI input is:

*Maximize X1*

*S.T.*

$$X1 + X2 > 0$$

$$2X1 + X2 < 2$$

$$X1 > 0$$

$$X2 < 0$$

*End*

*free X1*

*free X2*

Solution is ( $X_1 = 2$ ,  $X_2 = -2$ ) with optimal value of 2.

## 6. How to solve Integer LP?

Suppose in the Carpenter's Problem every table needs four chairs; then the LP formulation is:

$$\text{Max } 5X_1 + 3X_2$$

$$\text{S.T. } 2X_1 + X_2 \leq 40, X_1 + 2X_2 \leq 50, 4X_1 - X_2 = 0, X_1 \geq 0, X_2 \geq 0$$

*End*

*GIN X1*

*GIN X2*

GIN stands for general integer variable. The optimal solution is ( $X_1 = 5$ ,  $X_2 = 20$ ) with optimal value of 85.

Special case of binary variables ( $X = 0$  or  $1$ ) is also permitted in LINDO, the command to make the variable  $X$  a binary variable is `INT X1`.

## 7. How to solve Binary (0, 1) LP?

Suppose there are  $n$  items to be considered for inclusion in a Knapsack. Each item has certain per unit value to the traveler who is packing the knapsack. Each item has a per unit weight that contributes to the overall weight of the knapsack. There is a limitation on the total weight that can be carried. The objective is to maximize the total value of what is packed into the knapsack subject to the total weight limitation. We can use Binary LP to solve this problem.

Using the `INT` command in LINDO restricts a variable to being either 0 or 1. These variables are often referred to as binary variables. In many applications, binary variables can be very useful in modeling all-or-nothing situations. Examples might include such things as taking on a fixed cost, building a new plant, or buying a minimum level of some resource to receive a quantity discount.

Consider the following Knapsack Problem

$$\text{Maximize } 11X_1 + 9X_2 + 8X_3 + 15X_4$$

Subject to:

$$4X_1 + 3X_2 + 2X_3 + 5X_4 \leq 8,$$

and any  $X_i$  is either 0 or 1.

Since is this very small size problem there are 4 variables each can have either of 2 values, there are  $2^4 = 16$  possibilities. Trying all 16 possibilities in order to identify an optimal solution (if it exists) is tedious. Therefore, one must use any one of ILP software packages to solve even this or any larger-scale problem.

Using LINDO, the problem statement is:

```

Max 11X1 + 9X2 + 8X3 + 15X4
S.T. 4X1 + 3X2 + 2X3 + 5X4 ≤ 8
END
INT X1
INT X2
INT X3
INT X4

```

Then click on SOLVE. The output shows the optimal solution and the optimal value after 8 Branch-and-Bound Iterations.

Note that instead of repeating INT four times, one can use INT 4. The first four variables appeared in the objective function. The relevant output is:

```

OBJECTIVE FUNCTION VALUE
1)      24.00000

VARIABLE      VALUE
X1            0.000000
X2            1.000000
X3            0.000000
X4            1.000000

```

### 8. **Warning:** Shadow Price Might Have Wrong Sign

Some LP software packages do not obey strict duality for both maximization and minimization. Therefore one has to take that into account adjust the signs accordingly. This can be done by changing the RHS by “small” amount and finding the new optimal value, then using the definition of shadow price as the rate of change in the optimal value with respect to the change in the RHS.

For example, considering the following LP with a unique optimum solution:

Minimize  $18X_1 + 10X_2$   
Subject to:  $12X_1 + 10X_2 \geq 120000$   
 $10X_1 + 15X_2 \leq 150000$   
 $X_1, X_2 \geq 0$

Running this problem by LINDO, the final report gives the shadow prices  $U_1 = -2.125$ , and  $U_2 = 0.75$ , while the correct ones are  $U_1 = 2.125$ , and  $U_2 = -0.75$ .

### **Another example:**

Minimize  $3X_1 - 5X_2$   
S.T.  
 $2X_1 + X_2 \leq 40$   
 $X_1 + 2X_2 \leq 50$   
 $X_1, X_2 \geq 0$

Solving the problem, we get  $(X_1 = 0, X_2 = 25)$ , with optimal value of  $-125$ , and shadow prices  $(0, 2.5)$ . The correct shadow prices are  $(0, -2.5)$ .

### **9. Conclusions:**

We have walked through the use of the LINDO optimization software available at Citrix. We have tested its essential functionalities including saving files and printing to a local computer. It is easy to use and provides useful information. Its learning curve is very short (less than 30 minutes, presenting to students). The use of its functionalities Enhance learning managerial concept as a *computer assisted learning*. This software is the student version that is an added value because students will use the *profession* version; that is being used in *Commercial/Industry/Government*.