**Some Useful Nonlinear Optimization:**

1. Find two positive numbers whose sum are 100 and whose product is a maximum.

Maximize X.Y

Subject to:

X + Y = 100

X  0

Y  0

Solution:

X / 1 = Y /1

This gives

X = Y = 50

Therefore the optimal solution is (X = 50, Y = 50) with optimal value of X.Y = 2500.

2. Find two nonnegative numbers whose sum is 9 and so that the product of one number and the square of the other number is a maximum.

Maximize X.Y2

Subject to:

X + Y = 9

X  0

Y  0

Solution:

X / 1 = Y /2, Y = 2X

This gives

X + 2X = 9, 3X = 9,

Therefore,

X = 3, Y = 6 is the optimal solution with optimal value of

X. Y2 = 3(36) = 108.

3. Find two positive numbers such that their product is 25 and their sum is a minimum. What is the minimum sum?

Minimize X + Y

Subject to:

X Y = 25

X  0

Y  0

Solution:

X / 1 = Y /1,

This gives

XY = X2 = 25, X = 5, the non-negative root.

Therefore,

X = 5, Y = 5 is the optimal solution with optimal value of

X + Y = 5 + 5 = 10.

4. What about

Maximize XY

Subject to:

2X + 3Y = 60

X  0

Y  0

Let A = 2X, B = 3Y

X = A/2, Y = B/3

Substituting

Maximize AB/6

Subject to:

A + B = 60

A  0

B  0

The maximum occurs at A = B = 30

Therefore X = 15, and Y = 10 with optimal value of XY = 150.

**A Statistical Application**: What Binomial distribution has largest variance?

Variance npq

Where

p + q =1

p  0

q  0

Where n is number of trials.

**Solution:**  The variance is the worst for the Binomial with flat probability, i.e.

p = q = 1 / 2, in that case the largest variance is 0.25n.

**Notice that** the above results can be generalized for more than 2 decision variables. Moreover, unlike Linear Program, **the optimal solution may not be one of the vertices.**