**Review of Your Probability and Statistics with Financial Portfolio Selection**

**1. The Expected Value (i.e., averages, mean)**:

Expected Value =  =  (Xi  Pi), the sum is over all i's.

It is an important statistic, because, your customers want to know what to expect from your product/service

OR as a purchaser of raw material for your product/service you need to know what you are buying, in other

word what you expect to get:

To read-off the meaning of the above formula, consider computation of the average of the following data

2, 3, 2, 2, 0, 3

The average is Summing up all the numbers and dividing by their counts:

(2 + 3 + 2 + 2 + 0 + 3) / 6

This can be group and re-written as:

[2(3) + 3(2) + 0(1)] / 6 = 2(3/6) + 3(2/6) + 0(1/6) = =  (Xi  Pi),

That is the sum of each distinct observation times its probability. Right?

**2. The Variance is:** 2 = Var(X) = E[(X- )2] =  [Xi2  Pi] - 2,     the sum is over all i's.

**3. Coefficient of Variation**: Coefficient of Variation (CV) is the *relative deviation* with respect to size Description: http://home.ubalt.edu/ntsbarsh/Business-stat/xbaru.gif

provided Description: http://home.ubalt.edu/ntsbarsh/Business-stat/xbaru.gifis not zero, expressed in percentage:

CV =100 Description: http://home.ubalt.edu/ntsbarsh/Business-stat/xbaru.gif%

**1/4**

**An Application:** Consider the performance of following two investment alternatives

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **- Two Investments -** | | | | |
| **Investment I** | |  | **Investment II** | |
| Payoff % | Prob. |  | Payoff % | Prob. |
| 1 | 0.25 |  | 3 | 0.33 |
| 7 | 0.50 |  | 5 | 0.33 |
| 12 | 0.25 |  | 8 | 0.34 |

Using the [Multinomial](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/multinomial.htm" \t "new) (<http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/multinomial.htm>) for

calculation, we notice that the Investment I has **mean = 6.75%** and **standard deviation = 3.9%**, while the

second investment has **mean = 5.36%** and **standard deviation = 2.06%.** First observe that under the usual

mean-variance analysis, these two investments cannot be ranked. This is because the first investment has

the greater mean; it also has the greater standard deviation; therefore, the **Standard Dominance Approach**

is not a useful tool here. We have to resort to the coefficient of variation (C.V.) as a systematic basis of

comparison. The C.V. for Investment I it is **57.74%** and for Investment II is **38.43%.** Therefore,

Investment II has preference over the Investment I. Clearly this approach can be used to rank any number

of alternative investments. Notice that less variation in return on investment implies less risk.

Suppose **you wish to diversify** your investment between the two, the question is what percentages (W1) you should allocate to I, and the reset (1 –W1) to II?

You may use the coefficients of variation, inversely (why) related to W1, and 1-W2:

W1 / (1-W1) = 38 / 58 (Rounded). This gives W1 = 0.4 (that is 40%, to allocated to the first), and

1 – W1 = 0.6 (that is 60%, the rest on the second one).

**Question for you: What are the expected payoff and its quality of your diversified decision?**

**Hints**: If two random variable X and Y are independent, then the new random variable

Z = W1 X + W2 Y will have the expected value and variance:

E (Z) = E [ W1 X + W2 Y ] = W1 E(X) + W2 E(Y) = (0.4)(6.75%) + 0.6(5.36%) = 2.7 + 3.216 = 6%

And Variance of Z = Var (Z) = Var [ W1 X + W2 Y ] = W1 **2** Var(X) + W2 **2** Var(Y) = (0.4 2 )( 3.9 2 ) + (0.6 2 )( 2.062 ) = 2.4336 + 1.5277 = 3.9613, the square root is the standard deviation, i.e, 1.99

Coefficient of Variation of random variable Z is 1.99/6 = 33%

**2/4**

**Application of Signal-to-Noise Ratio In Investment Decisions:** Suppose you have several portfolios, which are almost uncorrelated (i.e., all paired-wise covariance's are almost equal to zero), then one may distributed the total capital among all portfolios proportional to their signal-to-noise ratios.

For [Negatively Correlated](http://home.ubalt.edu/ntsbarsh/Business-stat/opre/partVI.htm#ranotherExam) portfolios you may use [the Beta Ratio](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/MultiVariate.htm), or [Bivariate Discrete Distributions](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/Bivariate.htm) Javascript.

Consider the above two independent investments with the given probabilistic rate of returns. Given you wish to invest $12,000 over a period of one year, how do you invest for the optimal strategy?

The C.V. for Investment-I is 57.74% and for investment-II is 38.43%, therefore **signal-to-noise ratio** are 1/55.74 = 0.0179 and 1/38.43 = 0.0260, respectively.

Now, one may distribute the total capital ($12000) proportional to the Beta values:

Sum of signal-to-noise ratios = 0.0179 + 0.0260 = 0.0439

Y1 = 12000 (0.0179 / 0.0439) = 12000(0.4077) = $4892, Allocating to the investment-I

Y2 = 12000 (0.0260 / 0.0439) = 12000(0.5923) = $7108, Allocating to the investment-II

That is, the optimal strategic decision based upon the signal-to-noise ratio criterion is: Allocate $4892 and $7108 to the investment-I and investment-II, respectively.

These kinds of mixed-strategies are known as **diversifications** that aim at reducing your risky.

The quality of your decision may be computed by using [Performance Measures for Portfolios](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/RiskMeasur.htm).

**Extensions:** Now, consider three Investments choices with Coefficient of Variation (C.V.) of 1, 2 and 3, respectively, suppose you wish to diversify the total of $11, 000. How do you distribute your capital?

Let W1, W2, and W3 be the amount you allocate to each Investment respectively. The best policy is to invest inversely proportion to their C.V.’s. That is

W1 =t/1, W2 = t/2, and W3 = t/3,

Since

W1 + W2 +W3 = 11,000

Substituting for W’s in terms of t, we have”

t/1 + t/2 + t/3 =$11000

(6t + 3t + 2t) / 6 = 11,000

**3/4**

11t = 66,000, the factor t is t = 6,000

Thus one must allocate as follows:

W1 = $6,000/1 = $6,000, W2 = $6,000/2 = S3, 000, W3 = $6,000/3 = $2,000

**Diversification for the Classical Decision Analysis:**

**State of Economy**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Growth | Medium Growth | No Change | Low |  | **Risk Asses-** | **-ment** |
|  | G(0.4) | MG(0.3) | NC(0.2) | L(0.1) | Expected Value | Standard Dev. | C.V. |
| B | 12 | 8 | 7 | 3 | 8.9 | 2.9\* | 32% \*\* |
| S | 15 | 9 | 5 | -2 | 9.5 \* | 5.4 | 57% |

Now, consider the above two Investments choices with Coefficient of Variation (C.V.) of 32, and 57, respectively, suppose you wish to diversify the total of $72, 000. How do you distribute your capital?

Let W1, and W2 be the amount you allocate to each Investment respectively. The best policy is to invest inversely proportion to their C.V.’s. That is

W1 =t/32, and W2 = t/57,

Since

W1 + W2 +W3 = $72, 000

Substituting for W’s in terms of t, we have”

t/32 + t/57 = $72, 000

(57t + 32t ) / 1824 = $11,000

89t = $131328000, the factor t is t = 147,559,5.506

Thus one must allocate as follows:

W1 = 147,559,5.506/ 32 = $46,112, and W2 = $147,559,5.506/57 = $25,888

**4/4**