A Review of Statistics and Probability for

Business Decision Making under Risk

Extracted from

<http://home.ubalt.edu/ntsbarsh/Business-stat/opre504.htm>

<http://home.ubalt.edu/ntsbarsh/opre640a/partIX.htm>

Statistical concepts and statistical thinking enable you to:

* solve problems in a diversity of contexts.
* add substance to decisions.
* reduce guesswork.

**Statistics:** The original idea of "statistics" was the collection of information about and for the "state". The word statistics drives directly not from any classical Greek or Latin roots, but from the Italian word for state.

**Application:** The debate on abortion: Abortion is a "premeditated decision to murder." Or, as a leading feminist declared, "don't put your law into my body."

**Public Opinion:**

## Step 1

A random sample

size n? **Step 3** **Inferential statistics Step 4 Decision**

**Step 2** p - 1.96s/√n , p + 1.96s/√n **Making**

**Descriptive statistics** with 95% confidence

Mean (p), standard deviation (s) margin of error 1.96s/√n

P(yes) = 60%

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Computational Tool:

[**http://Home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/Descriptive.htm**](http://Home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/Descriptive.htm)

The above figure illustrates the idea of [statistical inference](http://home.ubalt.edu/ntsbarsh/Business-stat/opre504.htm#rstatInferentia#rstatInferentia) from a random sample about the [population](http://home.ubalt.edu/ntsbarsh/Business-stat/opre504.htm#rPopulation#rPopulation). It also provides estimation for the [population's parameters](http://home.ubalt.edu/ntsbarsh/Business-stat/opre504.htm#rparamerts#rparamerts); namely the expected value, the standard deviation.

The uncertainties in extending and generalizing sampling results to the population are measures and expressed by probabilistic statements called Inferential Statistics. Therefore, probability is used in statistics as a measuring tool and decision criterion for dealing with uncertainties in inferential statistics.

Business statistics is a scientific approach to decision making under risk. Business Statistics provides justifiable answers to the following concerns for every consumer and producer:

1. What is your or your customer's, Expectation of the product/service you sell or that your customer buys? That is, what is a good estimate for ?
2. Given the information about your or your customer's, expectation, what is the Quality of the product/service you sell or that you customers’ buy. That is, what is a good estimate for ?
3. Given the information about your or your customer's expectation, and the quality of the product/service you sell or you customer buy, how does the product/service compare with other existing similar types? That is, comparing several 's, and several 's .

Probability is derived from the verb to probe meaning to "find out" what is not too easily accessible or understandable.

**Probability is for measuring the occurrence of an event**

**General Mathematics of Probability**

**1.** General Law of Addition: When two or more events will happen at the same time, and the events **are not** mutually exclusive, then:

P (X or Y) = P (X) + P (Y) - P (X and Y)

Notice that, the equation P (X or Y) = P (X) + P (Y) - P (X and Y), contains especial events: An event (X and Y) which is the intersection of set/events X and Y, and another

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event (X or Y) which is the union (i.e., either/or) of sets X and Y. Although this is very simple, it says relatively little about how event X influences event Y and vice versa. If P (X and Y) is 0, indicating that events X and Y do not intersect (i.e., they are mutually exclusive), then we have P (X or Y) = P (X) + P (Y). On the other hand if P (X and Y) is not 0, then there are interactions between the two events X and Y. Usually it could be a physical interaction between them. This makes the relationship P (X or Y) = P (X) + P (Y) - P (X and Y) nonlinear because the P(X and Y) term is subtracted from which influences the result.

The above law is known also as the Inclusion-Exclusion Formula. It can be extended to more than two events. For example, for three events A, B, and C, it becomes:

P(A or B or C) =

P(A) + P(B) + P(C) - P(A and B) - P(A and C) - P(B and C) + P(A and B and C)

Special Law of Addition: When two or more events will happen at the same time, and the events **are** mutually exclusive, then:

P(X or Y) = P(X) + P(Y)

General Law of Multiplication: When two or more events will happen at the same time, and the events **are** dependent, then the general rule of multiplicative law is used to find the joint probability:

P(X and Y) = P(Y)  P(X|Y),

where P(X|Y) is a conditional probability.

Multiplicative Law: When two or more events will happen at the same time, and the events **are** independent, then the special rule of multiplication law is used to find the joint probability:

P(X and Y) = P(X)  P(Y)

Conditional Probability Law: A conditional probability is denoted by P(X|Y). This phrase is read: the probability that X will occur **given that** Y is known to have occurred.

Conditional probabilities are based on knowledge of one of the variables. The conditional probability of an event, such as X, occurring given that another event, such as Y, has occurred is expressed as:

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P(X|Y) = P(X and Y)  P(Y),

provided P(Y) is not zero. Note that when using the conditional law of probability, you always divide the joint probability by the probability of the event after the word given. Thus, to get P(X given Y), you divide the joint probability of X and Y by the unconditional probability of Y. In other words, the above equation is used to find the conditional probability for any two **dependent** events.

The simplest version of the Bayes' Theorem is:

P(X|Y) = P(Y|X)  P(X)  P(Y)

If two events, such as X and Y, are **independent** then:

P(X|Y) = P(X),

and

P(Y|X) = P(Y)

**The Bayes' Law:**

P(X|Y) = [ P(X)  P(Y|X) ]  [P(X) P(Y|X) + P(not X)  P(Y| not X)]

Bayes' Law provides posterior probability [i.e, P(X|Y)] sharpening the prior probability [i.e., P(X)] by the availability of accurate and relevant information in probabilistic terms.

**Three Schools of Thoughts in Probability:**

* Classical
* **Frequencist**
* **Bayesian**

**Probability Assessment:** Consider the following decision problem a company is facing concerning the development of a new product:

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# States of Nature

Action High Sales (A) Med. Sales (B) Low Sales (C)

**P(A) = 0.2** **P(B) = 0.5** **P(C) = 0.3**

A1 (develop) 3000 2000 -6000

A2 (don't develop) 0 0 0

**Expected Value: Σ xi . P(X = xi)**,

**Example:** 1, 2, 1, 3, 3

**Conditional Probabilities:** **⇓**

What the Consultant What Actually Happened in the Past

Predicted A B C

Ap 0.8 0.1 0.1

Bp 0.1 0.9 0.2

Cp 0.1 0.0 0.7

For Example, 0.2 is probability that the consultant predicted Bp while C really happened, i.e. P(Bp | C) = 0.2

**What do we need is, e.g., P(C | Bp) = ? A Posterior Probability**

**A question for you:** A family has two children given on is a boy, what is the chance that the other one is a boy?

**The Expected Value (i.e., the Averages) is:**

Expected Value =  = Xi . Pi,     the sum is over all i's.

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The expected value alone is not a good indication of a quality decision. The variance must be known so that an educated decision may be made.

For risk, smaller variance values indicate that what you expect is likely to be what you get. Therefore, risk must also be used when you want to compare alternate courses of action. What we desire is a large expected return, with small risk. Thus, high risk makes a manager very worried.

**Variance**: An important measure of risk is variance which is defined by:

Variance = 2 =  [Xi2 . Pi] - 2,     the sum is over all i's.

Since the variance is a measure of risk, therefore, the greater the variance, the higher the risk. The variance is not expressed in the same units as the expected value. So, the variance is hard to understand and explain as a result of the squared term in its computation. This can be alleviated by working with the square root of the variance which is called the **Standard Deviation**:

Standard Deviation =  = (Variance) ½

For the dynamic decision process, the Volatility as a measure for risk includes the time period over which the standard deviation is computed. The **Volatility measure** is defined as standard deviation divided by the square root of the time duration.

What should you do if the course of action with the larger expected outcome also has a much higher risk? In such cases, using another measure of risk known as the **Coefficient of Variation** is appropriate.

Coefficient of Variation (CV) is the relative risk, with respect to the expected value, which is defined as:

Coefficient of Variation (CV) is the *absolute relative deviation* with respect to size xbaruprovided xbaruis not zero, expressed in percentage:

CV =100 |S/xbaru| %

Notice that the CV is independent from the expected value measurement. The coefficient of variation demonstrates the relationship between standard deviation and expected value, by expressing the risk as a percentage of the (non-zero) expected value. The inverse of CV (namely 1/CV) is called the Signal-to-Noise Ratio.

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**An Application:** Consider two investment alternatives, Investment I and Investment II, with the characteristics outlined in the following table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **- Two Investments -** | | | | |
| **Investment I** | |  | **Investment II** | |
| Payoff % | Prob. |  | Payoff % | Prob. |
| 1 | 0.25 |  | 3 | 0.33 |
| 7 | 0.50 |  | 5 | 0.33 |
| 12 | 0.25 |  | 8 | 0.34 |

**Performance of Two Investments**

To rank these two investments under the *Standard Dominance Approach in Finance*, first we must compute the mean and standard deviation and then analyze the results. Using the [Multinomial](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/multinomial.htm" \t "new) for calculation, we notice that the Investment I has mean = 6.75% and standard deviation = 3.9%, while the second investment has mean = 5.36% and standard deviation = 2.06%. First observe that under the usual mean-variance analysis, these two investments cannot be ranked. This is because the first investment has the greater mean; it also has the greater standard deviation; therefore, the **Standard Dominance Approach** is not a useful tool here. We have to resort to the coefficient of variation (C.V.) as a systematic basis of comparison. The C.V. for Investment I is 57.74% and for Investment II is 38.43%. Therefore, Investment II has preference over the Investment I. Clearly, this approach can be used to rank any number of alternative investments. Notice that less variation in return on investment implies less risk.

You might like to use [Multinomial JavaScript](http://home.ubalt.edu/ntsbarsh/Business-stat/otherapplets/multinomial.htm" \t "new) for checking your computation and performing computer-assisted experimentation to show.

1. Show that E[aX + b] = aE(X) + b.
2. Show that V[aX + b] = a2V(X).
3. Show that: E(X2)= V(X) + (E(X))2.
4. Show that E[X/Y]  E(X)/E(Y).
5. Show that E[X  Y]  E(X)  E(Y).
6. Show that Cov(X, Y) = E(XY) - E(X)E(Y).
7. Show that Cov(X, X) = V(X).

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