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ZERO IN FOUR DIMENSIONS: HISTORICAL PSYCHOLOGICAL, CULTURAL AND LOGICAL PERSPECTIVE HOSSEIN ARSHAM

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Introduction

The introduction of zero into the decimal system in 13th century was the most significant achieve in the development of a number system, in which calculation with large numbers became feasibl Without the notion of zero, the descriptive and prescriptive modeling processes in commerce, astronomy, physics, chemistry, and industry would have been unthinkable. The lack of such a sy is one of the serious drawbacks in the Roman numeral system. In addition, the Roman numeral system is difficult to use in any arithmetic operations, such as multiplication. The purpose of this article is to raise students, teachers and the public awareness of issues in working with zero by providing the foundation of zero form four different perspectives. Imprecise mathematical thinkin by no means unknown; however, we need to think more clearly if we are to keep out of confusior

Our discomfort with the concepts of zero (and infinite) is reflected in such humor as 2 plus 0 still equals 2, even for large values, and popular retorts of similar tone. A like uneasiness occurs in confronting infinity, whose proper use first rests on a careful definition of what is finite. Are we mo hesitant to admit to our finite nature? Such lighthearted commentary reflects an underlying awkwardness in the manipulation of mathematical expressions where the notions of zero and infi present themselves. A common fallacy is that, any number divided by zero is infinity. It is not sim problem of ignorance by young novices who have often been mangled. The same errors are commonly committed by seasoned practitioners, yea, and even educators! These errors frequen can be found as well in prestigious texts published by mainstream publishers.

Historical Perspective

Counting is as old as prehistoric man is, after he learned to count, man invented words for numb and later still, symbolic numerals. The numeral system we use today originated with the Hindus. were devised to go with the 10-based, or "decimal," method of counting, so named after the Latir word decima, meaning tenth, or tithe. The first popularizer of this notation was a Muslim mathematician, Al-Khwarizmi in the 9th century, however it took the new numbers about two cent to reach Spain and then to England in a book called Craft of Nombrynge.

The Two Notions of Zero: The notion of zero was introduced to Europe in the Middle Ages by Leonardo Fibonacci who translated from Arabic the work of the Persian (from Usbekestan provin scholar Abu Ja'far Muhammad ibn (al)-Khwarizmi. The word "algorithm," Medieval Latin 'algorisn is a contamination of his name and the Greek word arithmos, meaning "number, has come to represent any iterative, step-by-step procedure. Khwarizmi in turn documented (in Arabic, in the 7th century) the original work of the Hindu mathematician Ma-háviral as a superior mathematical construction compared with the then prevalent Roman numerals which do not contain the concep zero. When these scholarly treatises were being translated by European accountants, they trans 1, 2, 3,.. upon reaching zero, they pronounced, "empty", Nothing! The scribe asked what to write was instructed to draw an empty hole, thus introducing the present notation for zero.

Hindu and early Muslim mathematicians were using a heavy dot to mark zero's place in calculati Perhaps we would not be tempted to divide by zero if we also express the zero as a dot rather th the 0 character.

Babylonians also used a zero, approximately at the same time as Egyptians, before 1500 BC. Certainly, zero's application in our base 10 decimal system was a step forward, as logarithms of Napier and others brought into use.

While zero is a concept and a number, Infinity is not a number; it is the name for a concept. Infini cannot be considered as a number since it does not follow numbers' properties. For example, (in + 2) is not more than infinity. Since infinite is the opposite of finite, therefore whoever uses "infinit must first give an indication for what is finite. For example, in the use of statistical tables, such as t-table, almost all textbooks denote symbol of infinity ( ) for the parameter of any t-distribution wi values greater than 120. I share Cantor's view that "….in principle only finite numbers ought to b admitted as actual."

Aristotle considered the infinite, as something for which there is no exit in an attempt to pass thrc it. In his Physics: Book III, he wrote "It is plain, too, that the infinite cannot be an actual thing and substance and principle."

Many writers have given much attention to clarifying the nature of the "infinite": what is it, how ca know anything about it, etc. Many constructively minded mathematicians such as David Hilbert choose to emphasize that we can restrict ourselves to the finite and thereby avoid many of these problems: this is the so-called "finitary standpoint".

Psychological Perspective

Zero as a concept, was derived, perhaps from the concept of a void. The concept of void existed

Hindu philosophy and the Buddhist concept of Nirvana, that is: attaining salvation by merging int void of eternity. Ma-hávíral (born, around 850 BC) was a Hindu mathematician, unfortunately, no much is known about him. As pointed out by George Wilhelm Friedrich Hegel, "India, such a vas country, has no documented history." In the West, the concept of void and nothingness appeared in the works of Arthur Schopenhauer during the 19th century, although zero as a number has bee adapted much earlier.

The Arabic writing mathematicians not only developed decimal notation, they also gave irrational numbers, such as square root of 2, equal rights in the realm of Number. And they developed the language, though not yet the notation, of algebra. One of the influential persons in both areas wa Omar Khayyam, known in the west more as a poet. I consider that an important point; too many people still believe that mathematicians have to be dry and uninteresting.

Initially, there was some resistance to accepting this significant modification to the time-honored Roman notation. Among the trite objections to leaving Roman numerals for the new notation wa difficulty in distinguishing between the numeral 1 and 7. The solution, still employed in Europe, w use a cross-hatch to distinguish the numeral 7.

The introduction of the new system indisputably marked the democratization of mathematical computation by its simplicity and lack of mystery. Up to then the "abacus" was the champion. Ab was a favorite tool for a few and praised by Socrates. The Greek's emphasis on geometry (i.e., measuring the land for agricultural purposes, the earth, thus the world geography) so kept them perfecting number notation system. They simply had no use for zero.

Sacrilegious as it may sound on first impression, the notation of zero is at heart nothing more tha directional separator as in the case of a thermometer. It is, in actuality, "not there." For example, order to express the number 206, a symbol is needed to show that there are no tens. The digit 0 serves this purpose. Zero became a part of the Natural Numbers System in the last century whe Giuseppe Peano puts it in his first of five axioms for his number theory. One may think of an anal Zero is similar to the "color" black, which is not a color at all. It is the absence of color, while the Light contains all the colors.

Zero is the only digit, which cannot stand-alone. It is a lonely number, lonelier than one. It require some sort of companionship to give meaning to its life. It can go on the left. On the right. Or both ways! Or in the middle as part of a threesome. Witness "01", "10", or "102". Even "1000". A relationship with other numbers gives it meaning (i.e. it is a dependent number). By itself it is not

When we write 10, we mean 1 ten and 0 ones. In some number systems, it would be redundant mention the 0 ones, because zero means there are no objects there. Place value uses relative positions. So an understanding of the role of 0 as marking that a particular 'place' is empty is essential, as is its role of maintaining the 'place' of the other digits. The usage of zero here is mo a qualitative than quantitative. Therefore, it is called an operational zero.

Another colleagues who is writing a paper for symbolic logic on zero stated that "It seems to be i 'nothing' in addition/subtraction, but if it is nothing then how can it effect numbers in multiplicatio Also as to your comment on 2/0 being meaningless. I am wondering what the answer should be, can be more clearly defined), and why."

Here, my dear colleague has mixed the two distinct notions of zero: Zero as a number being use our numerical systems AND as a concept for 'nothing'. As a result of this mixed-up, he is "wonde at his own mental creature. We used to think that if we know one, we know the other. We are fin out that we must learn a great deal more about "AND".

Cultural Perspective

Judging from the treatment accorded to the concept of zero, we do practice a variety of avoidanc mechanisms rather than confront the imagery associated with this seemingly difficult concept.

In reciting one's telephone number, social security number, postal zip code or post office box, roo number, street number or any of a variety of other numeric nominals, we carefully avoid pronoun the digit "zero" and instead substitute "oh." One may say "it is caused by our desire to communic quickly, if we can say the same thing in one syllable, why not?" What about number seven, shoul find a substitute for this too?

In some parts of the world, the phrasing "naught" and "aught" are used but it is quite uncommon hear "zero." All the other digits are correctly enunciated with this one curious exception.

Is the presence of nothing (reflecting non-existence) different from the absence of something (reflecting non-availability) or the absence of anything (reflecting non-existence)? Zero is a symb "not there" which is different from "nothing" "Not there" reflects that the number or item(s) exists b they are not just available. "Nothing" reflects nonexistence.

Zero not only has the quality of being nothing, it is also a noun, verb, adverb, and an adjective as "zero possibility". "We zeroed in on the cause," means we had isolated all the possibilities, and h discovered the one remaining. In this use as a verb, zero equals one. However, "The result was a fat, zero," uses the noun to express the idea of results of "nothing". Here, zero has the quality of being there. Zero as an action appears in the Conservative Laws of physics.

Is zero a number? Consider the following scene:

Ernie: I've put a number of cookies in that Jar. You can have them if you give me your teddy.
Bert: Great While Ernie hands over the teddy and looks eagerly in the jar, said:
Bert "Wait a Minute There's No Cookies Here. You Said You Put a Number of Cookies in There"
Ernie: That's right, zero is a number.

Clearly some sort of an avoidance mechanism is in operation. It is as though the name itself invo a kind of anxiety perhaps associated with "nothingness", a kind of emptiness which humankind fi uncomfortable and prefers to avoid confronting. As with all such anxiety- provoking ideas, some imagery is substituted which provides a veneer to mask the disquieting emotional undertones of discomforting idea. Zero represents the amount of nothing.

Today zero has a meaning not just of a number, but as the bottom, or failure. He made no baske he made zero baskets — meaning he failed to score. Or he gave zero assistance.

If you are familiar with Numerology, you notice that there is no zero to work with in the numbers t correlate with the alphabet, strange? Not at all. The absence of zero may suggest that the Pythagorean who first developed the duality between numbers and letters were not aware of the notion. The notion of zero is much younger.

On the telephone keypad, zero has the honor of representing the operator. There is no zero in m games, such as plying cards (after all who wants to win zero!). Zero is placed at the end of the keypad on the computer and at the bottom of the keypad on the telephone. Is zero the beginning the end? Notice that on a calculator's keypad the numbers starts with the largest numbers on the and work their way down to zero. What about the o and 0 being right next to each other on the P keyboard? Numbers are located three places. First it is located on the keyboard keys with the ra 1, 2,…,0; this is the same order that phone keypad. Second, on the right of the keyboard is a calculator-like pad where zero is the last listed number. Finally, there is a list of functions key, however there is no F0 because that could translate into no function and what would be the poin having a key "without" function. There will always be questions about the true meaning and funct of zero. Is it the end or the beginning? What does ground zero mean? Some use it as starting po the military uses it as an ending point.

The resistance against zero can be noted even at the architectural level in buildings where the gı level is rarely denoted as the zeroth-level as it should be. However, for mathematicians it comes easily to label the floors of a building to include zero, for example, the Department of Mathematic building at the University of Zagreb in Croatia has floors numbered as -1, 0, 1, 2, and 3. In fact, tl not a particularity of one building but a common practice in modern buildings in Spain and in Spa speaking world such as Argentina. The feeling of comfort with zero in these countries could be dı the fact that the Islamic culture had more influence in Spain than any other European countries. (countries do have a special word to say 'ground floor' in a conversation, not using a "0 button" fo ground floor.

Other Apparent Cultural Difficulties with Zero: It may be considered frivolous hyperbole to sugges that the demise of the Roman Empire was due to the absence of zero in its number system, but (can only ponder the fate of our civilization given the difficulty our culture seems to have with the presence of zero in our number system.

The notion of zero brings another wearying and yet intriguing questions: Is our current century th 20^{th} century or the 21^{st} century? According to the Holy Scriptures (see, Matthew chapter 2), King Herod was alive when Jesus was born, and Herod died in 4 BC. Does that mean the millennium actually started in 1996?

Ordinal numbers, which the Gregorian calendar uses, indicate sequence. Thus "A.D. 1" (or the fi year A.D.) refers to the year that begins at the zero point and ends one year later. Think of a carpenter's ruler, if you will; the first inch is the interval between the edge and the one-inch mark. Thus, e.g., the millennium ended with the passing of the two-thousandth year, not with its incepti Cardinal numbers, which astronomers use in their calculations, indicate quantity. Zero is a cardin number and indicates a value; it does not name an interval. Thus "zero" indicates the division between B.C. and A.D., not the interval of the first year before or after this point. Continuing with example, put two rulers end to end: although there is a zero point, there is no "zero'th" inch.

As it stands now, we refer to years with ordinal numbers and to ages with cardinal numbers. Thu child less than a year old is usually said to be so many weeks or months old, rather than "zero ye old." If we changed over to this system for our calendar (referring to the age of our era, rather tha the order of the year), then there would be "zero years" for both A.D. and B.C.! That is to say, the twelve months before the birth of Christ and the first twelve months after the birth of Christ would the years 0 B.C. and A.D. 0 respectively.

The main confusion is between the notions of "time window length" and a "point in time". There is interval between 0 and 1. Considering whether this century is 2000 or 2001, depends on whethe look at a number as a points on time or a time interval. Years are intervals; numbers are points. Therefore, it is always a mistake to treat years as points. For example, consider the old arithmeti question: John was born in 1985 and Jane in 1986.

How much older is John than Jane? The answer, of course, can be anywhere from a few second two years, depending on when in those intervals the two people were born.

This is quite revealing of the cultural predilections of the time when the calendar was reorganizec first under the Julian scheme undertaken under the auspices of the Roman Emperor, Julius Caes after whom the month of July was named, and subsequently under the Gregorian calendar curre in use, which was devised during the reign of Pope Gregory. What is quietly yet magnificently revealed by this now-curious omission is the absence of the notion of zero in the numbering syst then in use. When the notion of zero was subsequently introduced in the west in the Middle Ages could hardly have been regarded as feasible to rewrite the entire calendar, if the debate occurred the first place. Clearly then, our ideas about numbers permeate our culture.

The Babylonians, and Chinese did not have a symbol for zero. The word zero comes from the Ar "al-sifer". Introduced to Europe during Italian renaissance in the 12th century by Leonardo Fibona (and by Nemorarius a less well-known mathematician) as "cifra" from which we have obtained ou present cipher, meaning empty space. Sifer in turn is a translation of Hindi word "sunya" meaning or empty. In Hindi "shunya" means zero. The terms aught, naught, and cipher are older names in English for zero symbol. In French "chiffre" means zero. It may also make you wonder that the w "cifra" in Russian means "written numbers." Similarly, "Ziffer" in German means one single written number; it is used in contrast to a single letter. Zero in German is called "Null". The ancient Egyp never used a zero symbol in writing their numerals. Therefore there was no function for a zero in writing their numerals. The two applications of the zero concept used by ancient Egyptian scribes were:

1) as a zero reference point for a system of integers used on construction guidelines,

and

2) as a value that resulted from subtracting a number from an equal number.

It is quite extraordinary that neither the Egyptians nor the Greek were able to create a symbol to represent zero, or nothingness. The conceptual difficulty may have been that the zero is somethi that must be there in order to say that nothing is there. The Hindu-Arabic numerals were used fo written calculations in the West not before the 12th century, when Arabic texts were translated int Latin.

Logical Perspective

Reading the seventh edition of a book on Management Science (Taylor [64]), I found the author dividing 2 by zero in the Simplex linear optimization tableau while performing a column ratio test, the stated conclusion, 2 ÷ 0 = infinity ( ). A typographical error? Confusion? Willful sin? A telepho call bringing the obvious error to the attention of the publisher for correction in future editions was with an astonishing return call from the editor of the text still insisting that 2 ÷ 0 = .

Although both the author and editor insist on this computational outcome, they nonetheless some decline to continue the Simplex calculation based on this result, contrary to the logic of their conclusion.

Questions I had were: How can you divide two by zero? Which number, when multiplied by zero, gives you 2?

Dividing by Zero Can Get You into Trouble: If we persist in retaining such errata in our education texts, an unwitting or unscrupulous person could utilize the result to show that 1 = 2 as follows:

$(a).(a) – a.a = a^2 – a^2$

for any finite a. Now, factoring by a, and using the identity
$(a^2 – b^2) = (a – b)(a + b)$ for the other side, this can be written as:

a(a-a) = (a-a)(a+a)

dividing both sides by (a-a) gives

a = 2a

now, dividing by a gives

1 = 2, Voila!

This result follows directly from the assumption that it is a legal operation to divide by zero becau – a = 0. If one divides 2 by zero even on a simple, inexpensive calculator, the display will indicate error condition.

Again, I do emphasize, the question in this Section goes beyond the fallacy that 2/0 is infinity or r demonstrates that one should never divide by zero [here (a-a)]. If one does allow oneself dividing zero, then one ends up in the Hell. That is all.

It seems apparent that the zero paradox should be broken into to areas: mathematical and physi Not only is the need to define zero, but infinity as well. For some it is not a question of whether it exists, but merely what the definite result is."

One must make a clear distinction between the abstract concepts and the concrete concepts as as their useful implications in modeling process of reality. Therefore, one must engage in investig mathematical knowledge, especially the relation between conceptual and applied (procedural) knowledge. The distinction between these knowledge types is possible at a theoretical, epistemological and terminological level. One may classified them according to their different approach to a given problem:

Applied knowledge: How to get from where one is to where one wants to go in a finite number of steps.

Conceptual knowledge: How to get from where one is to where one wants to go in a finite or an infinite number of steps, or a leap without any steps at all.

An example of conceptual knowledge would be
Where one is: natural numbers
Where one wants to go: the end of them
How: Infinite number of steps.

For the applied knowledge it would be
Where one is: natural numbers
Where one wants to go: the end of them
How: In a finite number of steps depends on what calculator you are using.

As you see, conceptuality is subjective while realization is objective. Most conceptuality is metaphysical; while reality is mostly physical. One must recall that: being definite has the propert being definable.

The origin of the fallacy that any number divided by zero is equal to infinity goes back to the work Bháskara, an Hindu mathematician who wrote in the 12th century that "3/0 = , this fraction, of whi the denominator is cipher is termed an infinite quantity". He made this false claim in connection w an attempt to correct the wrong assertion made earlier by Brahmagupta of India that A / 0 = 0.

Notice that by this fallacy one tries to define "infinity" in terms of zero. Unfortunately, similar pract seem to prevail to the present day. A similar fallacy exists for logarithms of zero which is believed many to be (negative) infinity.

Is Zero Either Positive or Negative? Natural numbers are positive integer numbers. One horse, t trees, etc. However, the arrival of zero caused the inevitable rise of the even more nefarious numbers: The negative numbers.

What about negative numbers? The negative sign is an extension of the number system used to indicate directionality. Zero must be distinguished from nothing. Zero belongs to the integer set o numbers. Zero is neither positive nor negative but psychologically it is negative. The concept of z represents "something" that is "not there," while zero as a number represents the lowest of all no negative numbers. For example, if a person has no account in a bank, his/her account is nothing there). If he/she has an account, he/she may have an account-balance of zero.

A high school teacher told me that "…In High school Algebra books they like to teach about num You know whole numbers, natural numbers, rational numbers, irrational numbers, and integers to name a few. The problem that I often run across is where does the zero fit in. For instance 'a pos integer', does this include zero? We know that whole numbers include 0, but it is a positive whole number…"

She is right, unfortunately some algebra books are confusing on categorizing zero in our numeric systems. However, the accepted and widely use categories for inclusion of zero as a positive nur is "non-negative integers", while for excluding it from positive integer the terminology "positive integers" is used. Similarly, for the real numbers involving zero, the following four categories: "positive", "negative", "non-negative" and "non-positive" are being used. The last two categories include zero, while the first two exclude zero, respectively. Therefore, as you see, the first two se are the subsets of the last two, respectively.

Talking to another high school teacher, he stated that ".. I always thought and believed that zero i neither positive nor negative. It's only when we used the book International Student (7th Ed., by l Hornsby, and Miller, Addison Wesley, 1999, page 6) that:

when they presented inverse property of addition

a + (-a) = 0

they wrote these:

Number	Additive Inverse
6	-6
-4	-(-4) or 4
2/3	-2/3
0	-0 or 0

This is rather confusing to me and to my students because I told them that zero is neither positiv negative, then why did these authors attach a negative sign on zero?

I looked at other books and I found another one Modern Algebra and Trigonometry (3rd Ed., by Elbridge Vance, 1995), that when he also presented Existence of Additive inverses (axiom 6A), ir of his statements he wrote: 0 = -0.

All these are confusing. It is also a difficult and uncomfortable situation when a knowledgeable teacher want to correct the textbook, and the students taking the textbook as the ultimate authori if it's a Bible. One may like to remind them by mentioning that the purpose of education is the crit thinking for oneself.

The additive inverse of any number is a unique number. Therefore, the additive inverse of 0 canr be " -0, or 0″. (Thanks goodness! they did not include, double zeroes -00, and 00, etc.)

Moreover, the additive inverse of zero is itself. This property of zero also characterizes the zero (no other number has such nice property).

Furthermore, zero is the Null element for addition. Any operation has a unique Null. The inverse Null element for any operation is itself. For example, the Null element for both multiplication and division operations is 1.

Is Zero an Even or Odd Number? If one defines evenness or oddness on the integers (either pos or all), then zero seems to be taken to be even; and if one only defines evenness and oddness o natural numbers, then zero seems to be neither. This dilemma is caused by the fact that the conc of even and oddness predated zero and the negative integers. The problem posed by this questi that zero is not to be really a number not that it is even or odd.

Most modern textbooks apply concepts such as “even” only to “natural numbers,” in connection v primes and factoring. By “natural numbers” they mean positive integers, not including zero. Thos who work in foundations of mathematics, though, consider zero a natural number, and for them tl integers are whole numbers. From that point of view, the question whether zero is even just does arise, except by extension. One may say that zero is neither even nor odd. Because you can picl even number and divide it in groups, take, e.g., 2, which can be divided in two groups of “1”, and can be divided in two groups of “2”. But can you divide zero? That’s why there are so many “questions.”

If you feel that the question if zero is an even number is of no practical value at all, let me quote t following news from the German television news program (ZDF) “Heute” on Oct. 1, 1977:

Smog alarm in Paris: Only cars with an odd terminating number on the license plate are admittec driving. Cars with an even digit terminating were not allowed to be driven. There were problems: the terminating number 0 an even number? Drivers with such numbers were not fined, because t police did not know the answer.

“Is zero odd or even? One of my students suggested a convention, i.e. a useful unproved mecha which makes her feel better, that zero is indeed Even! She offered two arguments:

A1: “Odd” numbers are spaced two apart. So are “even” numbers. Proceeding downward, 8,6,4,2,0,-2,-4 .. should all be considered Even. While odd numbers 9,7,5,3,1,-1,-3 … skip over z in a most stubborn manner.

A2: Let two softball teams play a game, with each player betting one dollar a run to the opposing team. Further presume that no runs are scored (due to beer consumption) and no extra innings a allowed because it got dark.

The final score is zero to zero. If a player is asked by his wife whether he won or lost, he would probably indicate that he “broke even”. As the old math teacher said: ” Proof? Why any fool can s that.”

These issues make themselves strongly felt in the classroom, textbook, in the frequent mishandli the notion of zero by the novice and professional alike and therefore recommend themselves to c attention. These are among many issues of how to teach these concepts to students at early age

Continuous data come in the forms of Interval or Ratio measurements. The zero point in an Inter scale is arbitrary. The different scales for measuring temperature all have a zero, yet each has a different value! For example, on a Celsius thermometer, zero is set at the temperature at which p water freezes at the sea level altitude. While zero degrees Fahrenheit is 32 ° degrees below free and finally absolute zero is the theoretical point at which molecular movement ceases. Therefore since the absolute temperature can be created in the laboratory, it is only a concept. So, here on must accept that the meaning of zero is relative to its context. Now the question is: does 80 ° deg Fahrenheit temperature implies it's twice as hot as when it's 40 ° degrees? The answer is a No. W not?

Recently one of my students asked me "I want to know what the opposite of zero is." Well, not everything has an opposite. The concept of opposite is a human invention in order to make the w manageable, there is no real opposite in nature. Is day opposite of night? Is male opposite of fem or they are complementary to each other? What is the opposite of color blue? Here we must be cautious when we ask about apposite of zero. The difference is between quality (which is a conc versus quantity (which a number). For example, what is "minus red?" or what is opposite of red? However, in the context of the real line, you can say that the opposite of zero is itself, while the opposite of +2 is -2 with respect to the origin point 0, as both have the same distance from the or while one in on its right-side and the other on the left-side. This definition is acceptable if you acc the opposite of left is the right. What is the opposite of 1/2? If you say, it's 2, then 0 has no oppos

Concluding Remarks

Unfortunately I find that the act of dividing by zero is not at all an uncommon practice. Many references in applied mathematics can be found committing this and other errors. And if educato profess division by zero as an appropriate mathematical practice, they should not be surprised to this error persist among their students just as the teachers themselves learned this practice from own teachers. You might think, as one of my colleagues from Eastern Europe believed that "… th Anglo-Saxons culture do not have a way with numbers." While respecting this opinion, unfortuna found that this error is not limited to a particular culture. In fact, it is the problem often initiated by educators worldwide. For example, in the textbook for Educacion Mathematica by Gracia, et al. [1989, page 138], which is widely used in Spanish speaking Schools of Education, you will find th the function $y = 1/(X^2 – 1)$, evaluated at X = -1 is 952380952. Where did this number come from? right question one might ask is who educates our educators?

Ball [7] interviewed 10 elementary and 9 secondary teachers, asking, "Suppose that a student as you what 7 divided by 0 is. How would you respond? Why is that what you'd say?" What she foun was that 1 of the 10 elementary teacher candidates could explain using the meaning of the terms gave the correct rule, 5 gave an incorrect rule, and 2 didn't know. 4 of the secondary candidates explain using the meaning of the terms and 5 only gave the correct rule, e.g.; "You can't divide by zero . . . It's just something to remember," but gave no further justification when probed. Some of teacher who only gave the correct rule was math majors.

In most Elementary Education programs for prospective teachers, such as the one at the Towson State University in Maryland, it is required to take four math courses, concepts of mathematics I a II, plus teaching mathematics in the elementary school, together with a supervised math-teaching experience session. While the standard is high, the main question is who educates our educators Adding to this, doubling the existing difficulties for the teachers, the school systems hiring a teach seems to be more concerned about "how he/she would handle violence in the classroom?" Unfortunately, it is a miserable story to tell.

There must be a conviction that mathematics teacher and researchers in mathematics education have much to learn from each other, especially at a time when the school and adult curricula are converging. Based on my experience, I offer the following three distinct headings:

- **Recruitment:** What can be done to encourage reluctant would-be mathematics teachers to tak the plunge?
- **Retention:** What support do they need to enable them to become sufficiently competent, confi and comfortable with mathematics so that they can teach it to others?
- **Re-training:** What is it like teaching mathematics without a strong background in mathematics

Unfortunately, mathematics has been fundamentally depersonalized to "something machines do" that the meaningful response is that we need always to emphasis that mathematics has little valu divorced from imagination. Machines will always do 'imaginationless' mathematics better than humans. But "mathematics imagination meld" is needed by society and it can become a fascinati subject for most children in the classroom.

Too many pupils now think that mathematics is boring. Mathematics can and must be made more more relevant, and more challenging, for pupils and for teachers. The use of Internet interactive technology in the classroom can add a new and precious variety. This variety can help to engage hold pupils' attention, and can raise the chances that the lesson will have been judged a success The new interactive technology can help to attract and retain teachers by making the whole proc more business-like, more efficient and more effective. However the provision of appropriate hard software and training remain expensive and intractable hindrances to progress.

There is a "math" video series [Harlan Meyer, Diamond Entertainment, 1996]. One is called Addi then Subtraction, Multiplication and, of course, Division. The division segment of the series starts misspelling the word quotient. Then the "star" of the video shows how to divide by using repeated subtraction; however, she asks "If I have 12 doggy bones and I take away 4 groups of 3 bones, h many will I have left?" She answers herself, "Right, four." But it was the "trick" she claimed for div by zero. Unfortunately, there are many instances like this, which sent your blood pressure throug roof. Zero is nothing. So just remember nothing INTO something is nothing. Teaching kids to cou fine, but teaching them what counts is best.

One may view "division" as a subtraction operation. When you write 20/5 = 4, what you really me that how many times you can subtract 5 from 20? And the answer is 4 times. That is why division the "inverse" operation for multiplication, which is an addition. That is, 5 x 4 = 20, means, adds to itself 5, 4 times, and you will get 20. So dividing by "0" has no meaning, because the question: ho many times you can subtract nothing from something? The question itself makes no sense. The dividing by zero is meaningless. Therefore, it does not make to ask further what is its result, whe it is indeterminate or not?

Zero is an important concept, so time should be spent establishing that from early age one has s understanding of zero; zero, nought, nothing – as ever, the language should be varied. In absenc a concept of zero there could have been only positive numerals in computation, the inclusion of z in mathematics opened up a new dimension of negative numerals. Zero, when used as a countin number (such as zero defects), means that no such objects are present. A concept and symbol t connotes nullity represents a qualitative advancement of the human capacity of abstraction. As always, concepts are only real in their correct context.

Unfortunately, there are teachers who continue misleading students as the following argument illustrates: "When we multiply 4 times 3, what we're really doing is adding 3 plus 3 plus 3 plus 3. in a sense, multiplication is just really fast addition, right? Well, as it turns out, division is just real fast subtraction. So, if you're diving 12 by 3, the answer is the number of times you can subtract from 12 before you get to zero (i.e., 12 – 3 – 3 – 3 = 0). So, the answer is 4. Now that you know t imagine what happens if you try to divide 12 by 0. You start subtracting zeroes, you realize that y are doing it infinite times. So, division by zero is infinity."

But when you start subtracting zeroes, even infinite times, you never get down to zero! One shou never divide by zero. Our high school curriculum should put more emphasis on mathematical modeling rather than maths which in most cases are merely "puzzle solving" which has nothing t with students' lives. This will bring excitement in learning the language of mathematics and its applications.

Notes, Further Readings, and References

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The use of comma's and points still remains a nuisance in understanding numbers. In the Englis speaking world 1,000 means a thousand in many other languages (such as Spanish) it means o on the other hand 1.000 is a thousand in some languages and only 1 in the English speaking wo
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The first underlines the mathematical suitability of 360 (its factors are 2, 3, 4, 5, 6, 8, 9 ,10, 12, et problems related to the division of a whole in equal parts, the second points out the connection w come astronomical constants (as 365).
The second thesis is the fact that the Babylonian had a sexagesimal system, which was used in Greek astronomy. The fact, that a year consists of little more than 360 days, seems to be seconc The Babylonians did have a calendar with 360 days per year, plus suitable "additional days". Act it is supported by a clear 'semantic' link (day=degree) and by some historical facts: for example Chinese astronomy had 365 and 1/4 degrees, the Babylonian ephemerides were based on mear synodic months divided in 30 parts and the year was divided in 12 parts, etc.
The sexagesimal system seems to have been a basis of ancient thinking. Their day measuremer was the development of a 24 hour system (spherically, each hour being one half of 30 degree segments relative to 360 degrees)… hours also divided into 60 minutes, minutes into 60 seconds Attempts to develop measurable systems of "time" added their own bit of complexity to what was already a complex and culturally variant attempt to juxtapose precision in calendar and time syst congruent with a celestial system which seemed to defy precision at the time.
Our desire for a mathematical modeling of the universe and its processing difficulties is apparent too. Some interesting analogous ones existed also in music, architecture, etc. These models req the fitting between small integer numbers, easy to be represented and dealt with, and complex phenomena whose numerical parameters did not exactly fit in the integer-based scheme. It is cre that the 360-system, and the 6-8-9-12 scheme in music, were the results of this conflict, being mathematically suitable and semantically justified.
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Note also that in classic languages the first few numbers were adjective (i.e. inflected for gender, number, case): 1, 2, 3, 4 in Greek, 1, 2, 3 in Latin. In the old Russian language when following 2, and all their compounds the noun is in the Genitive Singular however, when following 5, 6, 7, 8, 9 and all their compounds as well as 10 and 11 the noun is in the Genitive Plural. Also when follow 100 and its multiples.

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